1161 – Seçkin ARI | ari@sakarya.edu.tr

2017 Yazokulu BLNT6NBS Dersnotu
http://www.bulentaltinbas.com.tr/Isaretler_ve_Sistemler_6NBAS_DersNotu.pdf

2017 Yazokulu Örnek Vize Soruları
http://www.bulentaltinbas.com.tr/Isaretler_ve_Sistemler_6NBAS_OrnekVize_Sinav.pdf

2017 Yazokulu Vize-Quiz-Final
http://www.bulentaltinbas.com.tr/Isaretler_ve_Sistemler_6NBAS_QVF_Sinav.pdf

İşaret nedir?

Fiziksel bir sistemin davranışına ya da durumuna ilişkin bilgi taşıyan ve bir ya da daha fazla bağımsız değişkene bağlı olarak değişen her türlü büyüklüğe işaret diyoruz.

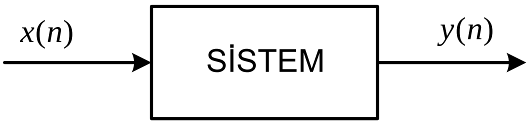
Örneğin: Fırın Sıcaklığını ayarlama; Giriş (voltaj değerini ayarlama) → Çıkış (çıkan sıcak hava), sensörler kullanılıyor, sensörlerin çıkışları sinyal olarak ifade ediliyor.

- ☐ Kararsız sistem (Sizin ayarladığınız değerlere ulaşmayabilir)
- ☐ Matematiksel yöntemin güvenirliği sağlaması gerekiyor.
- ☐ İşaretler zamanın bir fonksiyonudur

Örn.

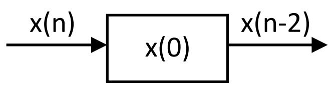
Arabaların hızlanması; arabanın hızı işaret

Su tankları; suyun akış hızı işaret



Sinyaller

1. Analog ve sayısal sinyaller
2. Gerçel ve karmaşık sinyaller
3. Gerekirci ve rassal sinyaller
4. Çift ve tek sinyaller
5. Periyodik ve periyodik olmayan sinyaller
6. Enerji ve Güç sinyalleri



$x[n]$ *veya* $x(n)$ **n:**zaman (tamsayı değerler alır)

$x[0]$ x dizisindeki 0.elemanı verir

$x[-1]$ -1 anındaki genlik değeri (herhangi sayısal bir değere sahip olabilir)

Time Shift (Öteleme)

$x[n] \rightarrow x[n-n_0] \quad n_0 > 0 \qquad x[n-8]$

$x[t] \rightarrow x[t-t_0] \quad t_0 < 0 \qquad t[t+5]$

$x(n-k)$ sağa öteleme/ Geçmiş hakkında bilgi

$x(n+k)$ sola öteleme / Gelecek hakkında bilgi

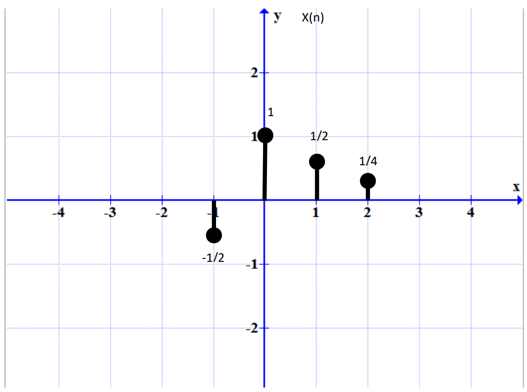
$x(-n)$ zaman ekseninde ters çevirme

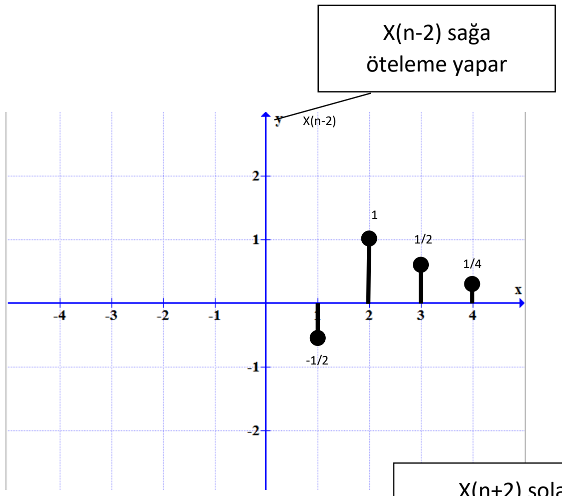
Örnek

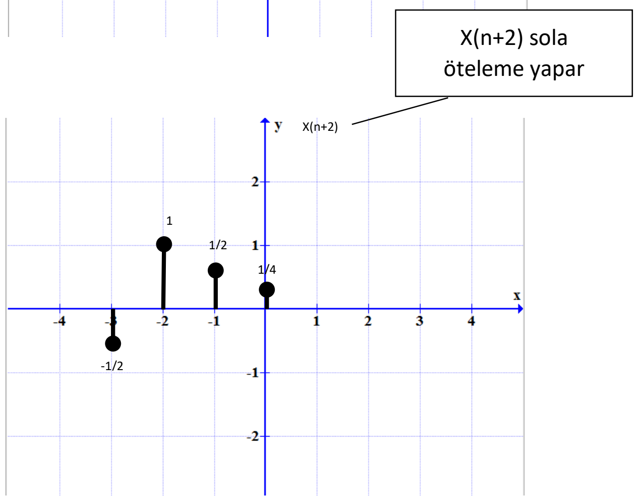
$x[n-8]$

$$n=0 \rightarrow x(-8)$$
$$\vdots$$
$$\vdots$$
$$n=8 \rightarrow x(0)$$

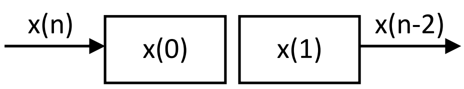
Örnek







Fark denklemlerinde bu ötelemeler kullanılarak ifade edilir.

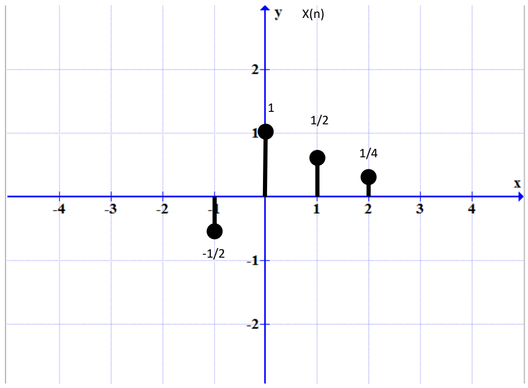


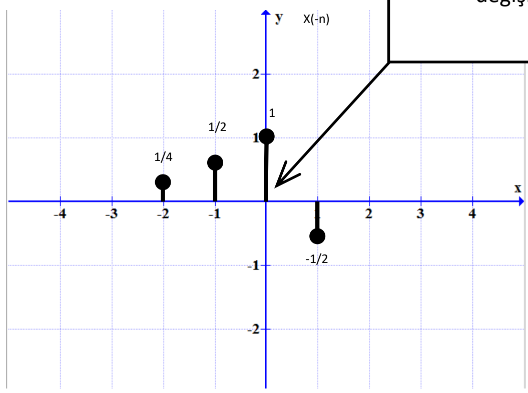
Bufferlama işlemi; İşaretin geçmiş değerlerine bu şekilde ulaşabiliriz

Time Reversal (Zamanı Ters Çevirme)

$$x[n] \qquad\qquad x[t]$$
$$\downarrow \qquad\qquad \downarrow$$
$$x[-n] \qquad\qquad x[-t]$$

Burada 0 anında bir değişiklik yok!





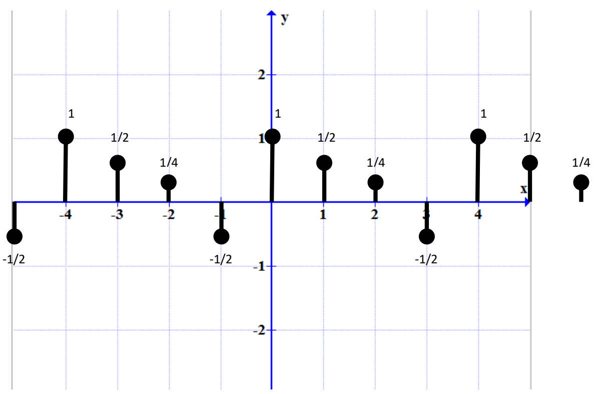
$$x[n] = x[-n] \qquad\qquad x[n-2] = x[-(n-2)] = x[-n+2]$$

Sola ötelemeye örnektir fakat –n olduğu için **sağa** ötelemedir!

İşaretin periyodik olması

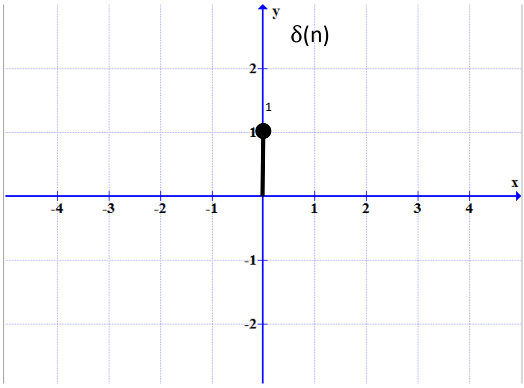
$$x[n] \quad = x[n+N]$$
$$x[0] \quad = x[N]$$
$$x[-1] \quad = x[N-1]$$

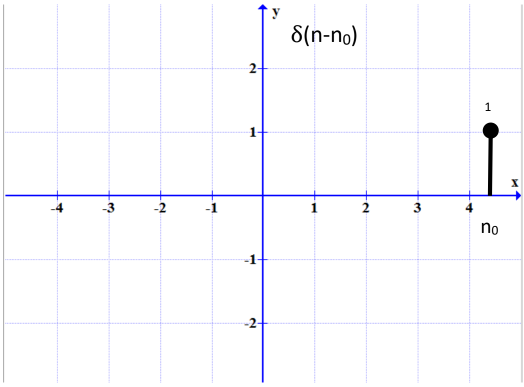
Bu koşul sağlanıyorsa; Ayrık işaretimizin periyodik olduğunu söylüyoruz ve N ile ifade ediyoruz
N tamsayı değerler alabiliyorsa periyodik.



Impulse işareti

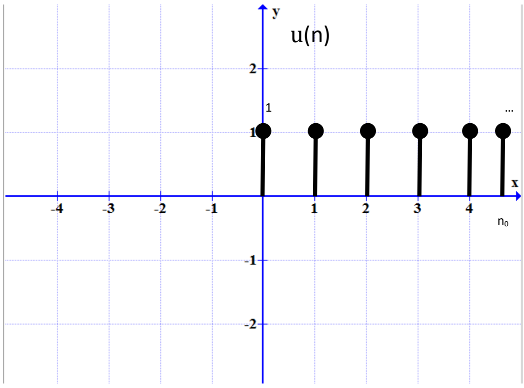
$$\delta[n]=\begin{cases}0 & n\neq 0\\ 1 & n=0\end{cases}$$

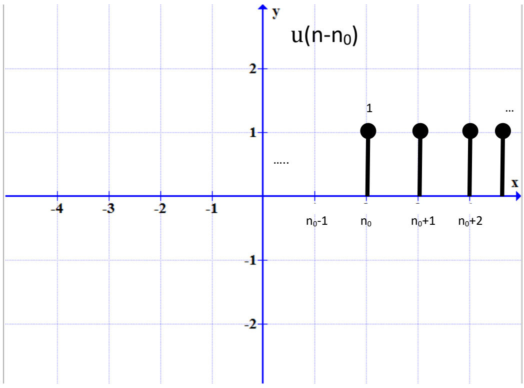




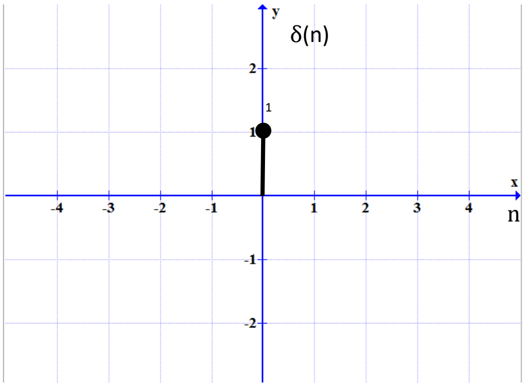
Birim basamak işareti

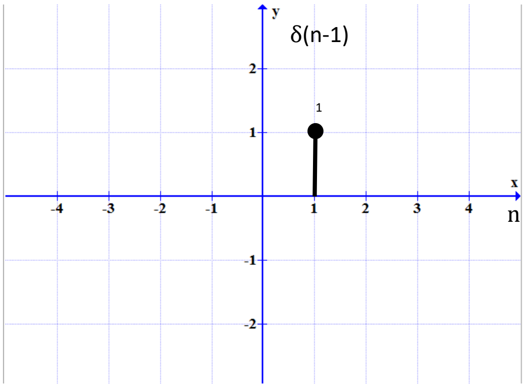
$$u[n]=\begin{cases}0 & n<0\\ 1 & n\geq 0\end{cases}$$

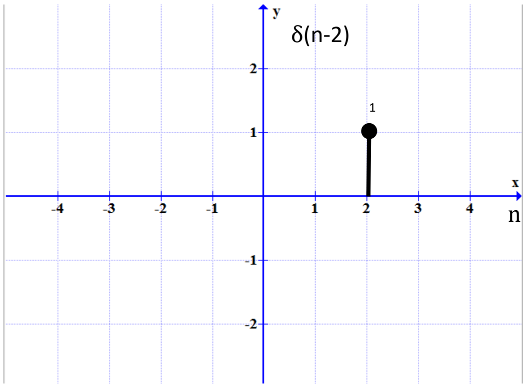


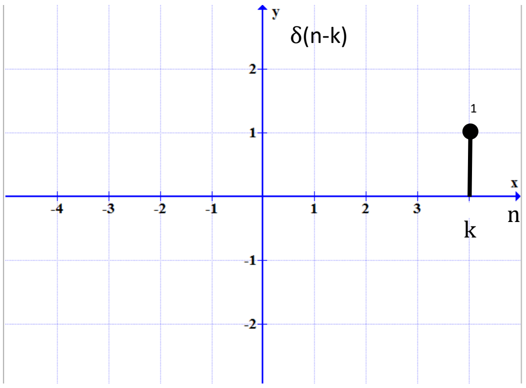


$\delta(n)$ *ve* $u(n)$ ***dönüşümleri***







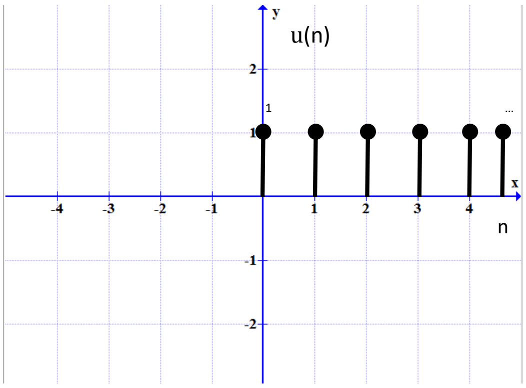


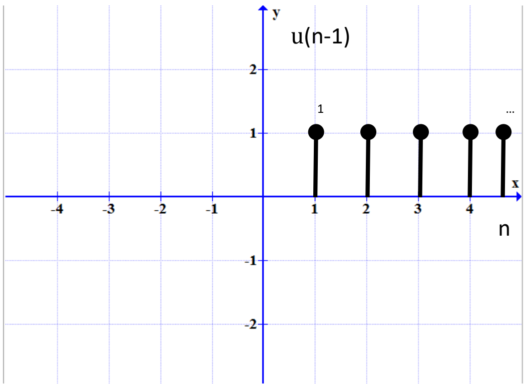
$$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

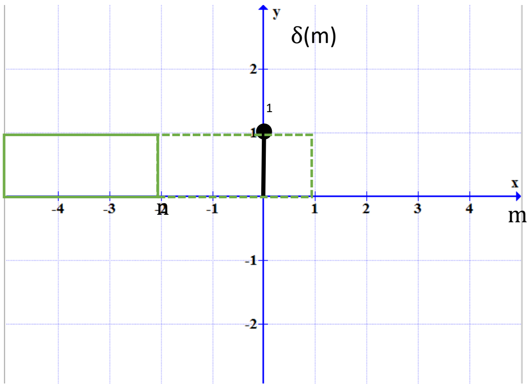
n ayrık, **t** analog ifade olduğunu gösteriyor

$$\delta[n] = u(n) - u(n-1)$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$







$$\sum_{m=-\infty}^{n} \delta[m] = 0 \quad n < 0$$

$$\sum_{m=-\infty}^{0} \delta[m] = 1 \quad n = 0$$

$$\sum_{m=-\infty}^{0} \delta[m] = 1 \quad n > 0$$

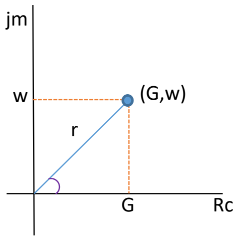
Üstel işaretler

$$x(n) = a^n u(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Karmaşık sayılar

$$Q + jw \quad \Rightarrow \quad \begin{cases} r = \sqrt{Q^2 + w^2} \\ \tan(\theta) = \dfrac{w}{Q} \end{cases}$$

$$Q + jw \quad \Rightarrow \quad re^{j\theta}$$



r =Genlik

$$\sigma(sigma) = r.\cos\theta$$

$$w = r.\sin\theta$$

$$\sigma + jw = r(\cos\theta + j.\sin\theta) = r.e^{j\theta}$$

$$f(\theta) = \cos\theta + j.\sin\theta$$

$$f'(\theta) = -\sin\theta + j\cos\theta$$

$$f'(\theta) = j.f(\theta)$$

Üstel fonksiyonun türevi kendisinin j ile çarpımına eşit ise üsteldir.

$$\begin{array}{rl} & e^{j\theta} = \cos\theta + j\sin\theta \\ + & e^{-j\theta} = \cos\theta - j\sin\theta \\ \hline & e^{j\theta} + e^{-j\theta} = 2.\cos\theta \end{array}$$

$$\begin{array}{rl} & e^{j\theta} = \cos\theta + j\sin\theta \\ - & e^{-j\theta} = \cos\theta - j\sin\theta \\ \hline & e^{j\theta} - e^{-j\theta} = 2j.\sin\theta \end{array}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad toplama$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \qquad çıkarma$$

Örnek $x(n)=e^{jw_0 n}$ bu işaret periyodik midir?

$$\begin{aligned} x(n) &= x(n+N) \\ e^{jw_0 n} &= e^{jw_0(n+N)} \\ e^{jw_0 n} &= e^{jw_0 n}\cdot e^{jw_0 N} \\ 1 &= e^{jw_0 N} \end{aligned}$$

$$2\pi k = w_0 N$$

$$N=\frac{2\pi k}{w_0}$$

k için tam sayı değerler varsa periyodik

$$x(n)=e^{jw_0 n}=\cos(w_0 n)+j\sin(w_0 n)$$

İşaretimizin frekansı

Örnek $x(n)=e^{j\frac{\pi}{4}n}$ bu işaret periyodik midir?

$$N=\frac{2\pi k}{w_0} \qquad N=\frac{2\pi}{\frac{\pi}{4}}\cdot k=8k$$

8 örnekte bir tekrar ediyor

$$x(0)=1$$
$$x(8)=1$$
$$x(16)=1$$
$$\vdots$$

Periyodik

Örnek $x(n)=e^{j\frac{\pi}{8}n}$ dizisi periyodik midir?

$$N=\frac{2\pi k}{w_0} \qquad N=\frac{2\pi}{\frac{\pi}{8}}\cdot k=16k \qquad \textit{Periyodik}$$

Örnek $x(n)=e^{j\frac{n}{8}}$ bu işaret periyodik midir?

$$N=\frac{2\pi k}{w_0} \qquad N=\frac{2\pi}{\frac{1}{8}}\cdot k=16\pi k \qquad \textit{Periyodik değil}$$

Örnek $x(n)=\cos(\frac{2\pi}{3}n)$ bu işaret periyodik midir?

$$N=\frac{2\pi k}{w_0} \qquad N=\frac{2\pi}{\frac{2\pi}{3}}\cdot k=3k \qquad \textit{Periyodik}$$

3 örnekte bir tekrar ediyor

$$x(0)=1$$
$$x(1)=-\frac{1}{2}$$
$$x(2)=-\frac{\sqrt{3}}{2}$$
$$x(3)=1$$
$$\vdots$$

$$x(n) = x_1(n) + x_2(n)$$

$$x_1(n) = N$$
$$x_2(n) = M$$

$$x(n) = x(n+L)$$

$$x_1(n) = x_1(n+mN)$$
$$x_2(n) = x_2(n+kM)$$

$$x_1(n) + x_2(n) = x_1(n+L) + x_2(n+L)$$

$$x_1(n+mN) + x_2(n+kM) = x_1(n+L) + x_2(n+L)$$

$$L = mN + kM$$

Örnek $x(n) = \cos(\frac{\pi}{3}n) + \sin(\frac{\pi}{4}n)$ periyodik midir?

$$x(n) = \cos(\frac{\pi}{3}n) + \sin(\frac{\pi}{4}n)$$

$$N = \frac{2\pi m}{w_0} = \frac{2\pi m}{\frac{\pi}{3}} = m.6$$

$$M = \frac{2\pi k}{w_0} = \frac{2\pi k}{\frac{\pi}{4}} = k.8$$

$L = mN + kM = m6 + k8$ *Periyodiktir.*

$$\cos^2\theta = \frac{1+\cos\theta}{2}$$

Örnek $x(n) = \cos^2(\frac{\pi}{8}n)$ periyodik midir?

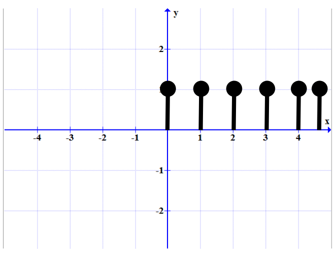
$$x(n) = \cos^2(\frac{\pi}{8}n) = \frac{1+\cos(\frac{\pi}{4}n)}{2} = \frac{1}{2} + \frac{1}{2}\cos(\frac{\pi}{4}n)$$

Frekans

$x(k) = 1$ $x(m) = 8$

$L = 1k = 8m = 8$

Katsayı genliği değiştirir



$$x(n) = x(n+N)$$
$$x(n) = x(n+1)$$

Ödev $x(n)=\cos(\frac{\pi}{8}n^2)$ periyodik midir? Cevap: Periyodiktir, 8

$$x(n)=x(n+N)$$

$$\cos\left(\frac{\pi}{8}n^2\right)=\cos\left(\frac{\pi}{8}(n+N)^2\right)$$

$$=\cos\left(\frac{\pi}{8}n^2+\underbrace{\frac{\pi}{8}2nN}_{2\pi nk}+\underbrace{\frac{\pi}{8}N^2}_{2\pi m}\right)$$

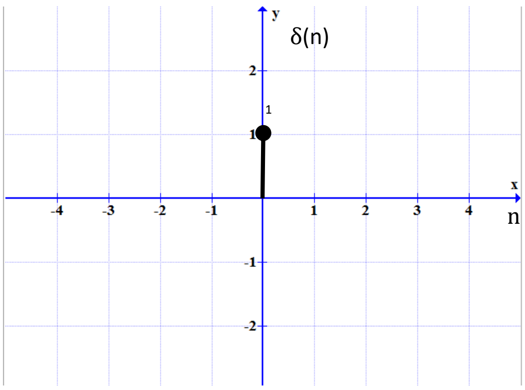
$$2\pi nk=\frac{\pi}{8}2\pi nN \qquad 2\pi m=\frac{\pi}{8}N^2$$

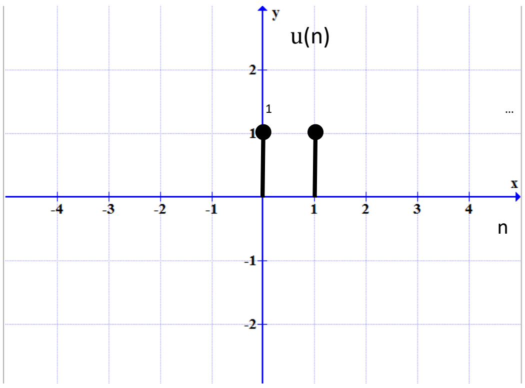
$$N=8k \qquad N^2=16m$$

Ötelenmiş birim işaretlerin toplamı

$$\delta[n]=u(n)-u(n-1)$$

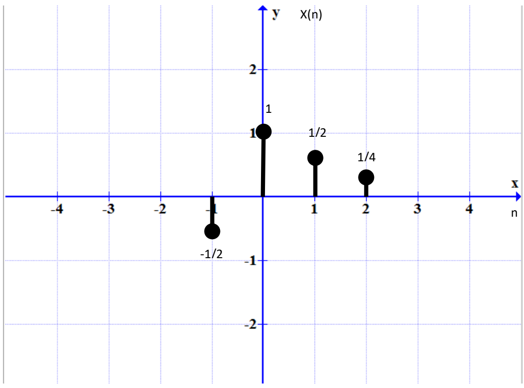
$$u[n]=\sum_{k=0}^{\infty}\delta(n-k)$$



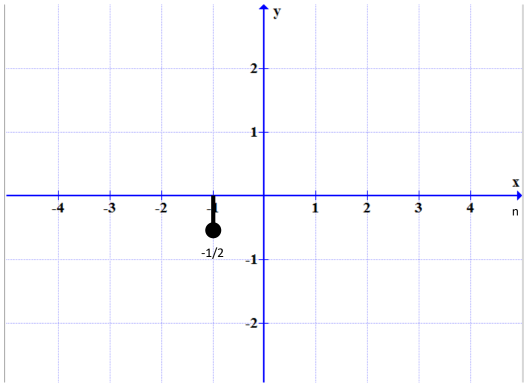


$$x(n)=?$$

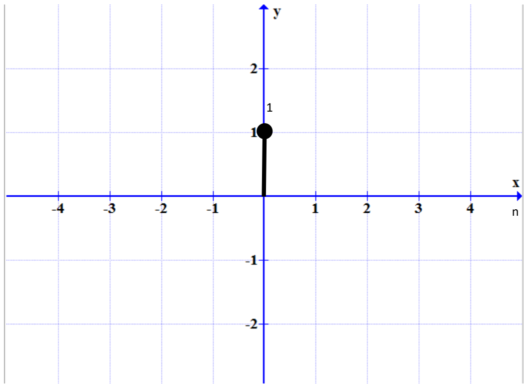
Örnek



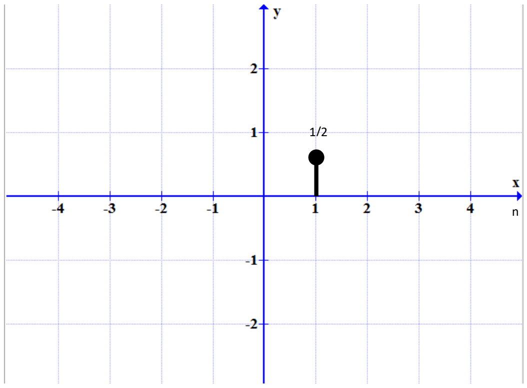
Çözüm



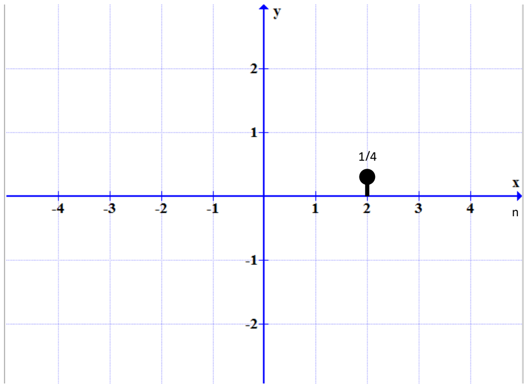
$$x(-1).\delta(n+1) = -\frac{1}{2}\delta(n+1)$$



$$x(0).\delta(n) = 1.\delta(n)$$



$$x(1).\delta(n-1) = \frac{1}{2}.\delta(n-1)$$



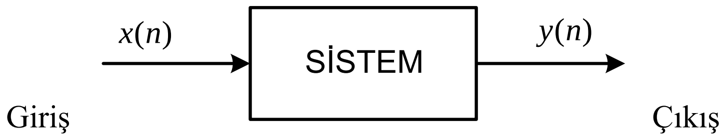
$$x(2).\delta(n-2) = \frac{1}{4}.\delta(n-2)$$

$$+\overline{\quad x(-1).\delta(n+1) + x(0).\delta(n) + x(1).\delta(n-1) + x(2).\delta(n-2)}$$

Genel İfade

$$x(n) = \sum_{k=-\infty}^{\infty} x(k).\delta(n-k)$$

DOĞRUSAL ZAMANDA DEĞİŞMEYEN SİSTEMLER



$$y(n) = T[x(n)]$$

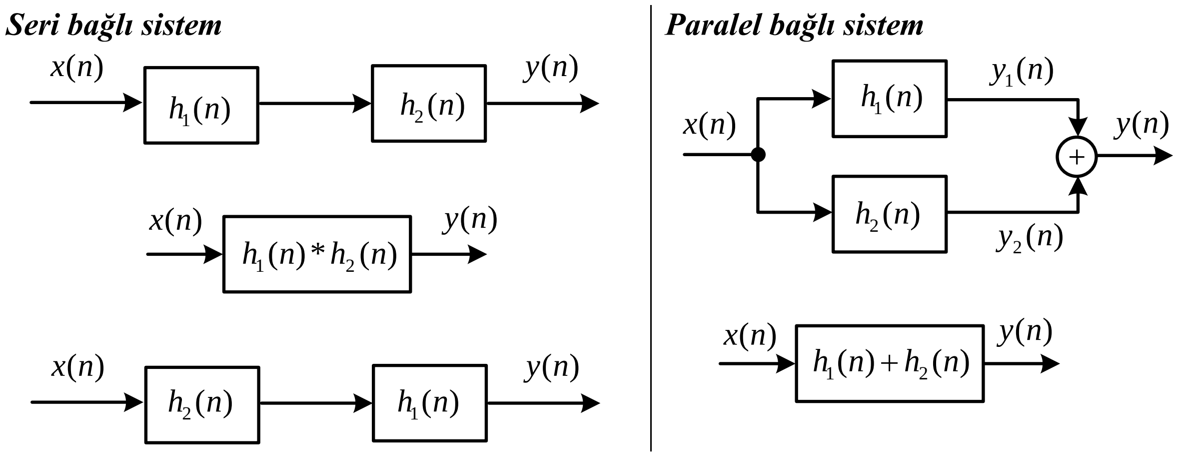
Örnek Banka faizi hesaplayan sistem

$$\left.\begin{aligned} y(n) &= 1.01 \cdot y(n-1) + x[n] \\ y(n) - 1.01 \cdot y(n-1) &= x[n] \end{aligned}\right\} y[n] + ay[n-1] = bx[n]$$

Örnek Fark denklemleri, Diferansiyel denklemler

$$x(0) = y(0)$$
$$y(1) = y(0) + 0,01 \cdot y(0) = y(0) \cdot 1,01$$
$$y(2) = y(1) + 0,01 \cdot y(1) = y(1) \cdot 1,01 + x(2)$$
$$\vdots$$
$$\vdots$$
$$y(n) = y(x-1) \cdot 1,01 + x(n)$$

Örnek Ses sistemleri



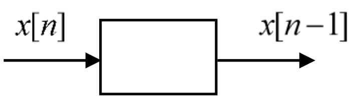
Ayrık zamanlı sistemler ve özellikleri

1. Hafızalı
2. Nedensel (Gerçeklenebilirlik)
3. Kararlılık
4. Zamanla değişmezlik
5. Doğrusallık

1. Hafızalı

$$y[n]=\left(2x[n]-x[n]^2\right)^2$$

$$y(t)=Rx(t)$$



$$y[n]=\sum_{k=-\infty}^{n} x[k]$$

$$y(n)=x[n-1]$$

$$y(t)=\frac{1}{c}\int_{-\infty}^{\tau} x(\tau)d\tau$$

Önceki değeri çıkışa gönderiyor.

Sistem (sola ve/veya sağa) ötelenmiş hallerden birine sahipse **hafızalı**dır.

2. Nedensel (Gerçeklenebilirlik)

x bilgisinin sadece geçmişteki bilgisine sahipsek; **n** ve/veya **(n-k)** gibi sağa ötelenmiş hallerinden biri ise **nedensel**, sistemin nedensel olması gerçeklenebilir olması demektir.

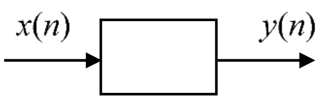
3. Kararlılık

Zaman ekseninde çıkış işareti belli bir sınır aralığında kalıyorsa; Girişse uygulanan işaretin genliği sınırlı bir aralıkta değişiyorsa, girişe uygulanan işaret ile sistem çıkışı da belirli bir aralıkta duruyorsa sistem **kararlıdır**.

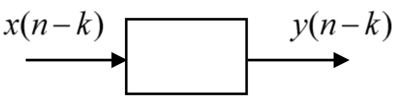
Örnek Banka faiz örneğinde sistem kararsızdır.Giren para max 1000 olsa bile çıkan hesaptaki para ∞'a gidebilir.

4. Zamanla değişmezlik

Giriş miktarına uygulanan öteleme sistemin çıkışında da aynı öteleniyorsa **zamanla değişmez.**



$$y(n)=T[x(n)]$$



$$x_1(n)=x(n-k)$$

$$y_1(n)=T[x_1(n)]=T[x(n-k)]$$

Koşul: $y_1(n)=y(n-k)$ olması gerekir.

Örnek: $y(n)=8n\,x(n)$ sistemin özellikleri hakkında bilgi veriniz?

$$y(n)=8n\,x(n)$$

$$T[x(n-k)]=8n\,x(n-k)$$

$$x_1(n)=x(n-k)$$

$$eşit\ değil\left\langle \begin{array}{l} y_1(n)=8n\,x_1(n)=8n\,x(n-k) \\ y(n-k)=8(n-k)\,x(n-k) \end{array}\right.$$

- (n-k) yok Hafızalı Değil
- x(n) var Nedensel
- $y_1(n)\neq y(n-k)$ eşit olmadığı için;
  Zamanla değişmez değil (Sabit n var)
- Kararsız n ∞'a gidiyor

Örnek: $y(n)=T[x(n)]=x(n)+4x(n-3)$ sistemin özellikleri hakkında bilgi veriniz?

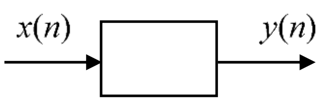
$$y(n)=T[x(n)]=x(n)+4x(n-3)$$

$$x_1(n)=x(n-k)$$

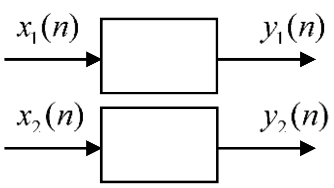
$$eşit\left\langle \begin{array}{l} y_1(n)=x_1(n)+4x_1(n-3)=x(n-k)+4x(n-k-3) \\ y(n-k)=x(n-k)+4x(n-k-3) \end{array}\right.$$

- (n-k) var Hafızalı
- x(n) ve x(n-k) var Nedensel
- $y_1(n)=y(n-k)$ eşit olduğu için
  Zamanla değişmez
- Kararlı n ∞'a gitmiyor

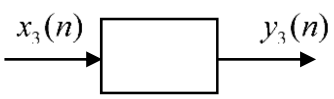
5. Doğrusallık



$$y(n)=T[x(n)]$$



$$x_3(n)=ax_1(n)+bx_2(n)$$



$$y_3(n)=T[x_3(n)]=T[ax_1(n)+bx_2(n)]$$

$$T[x(n)]=y(n)$$
$$T[x_1(n)]=y_1(n)$$
$$T[x_2(n)]=y_2(n)$$
$$x_3(n)=ax_1(n)+bx_2(n)$$
$$T[x_3(n)]=y_3(n)$$
$$y_3(n)=ay_1(n)+by_2(n)$$

<u>Sistem Doğrusal ise;</u>

$$y_3(n)\overset{?}{=}T[ax_1(n)]+T[bx_2(n)]$$

$$y_3(n)\overset{?}{=}aT[x_1(n)]+bT[x_2(n)]$$

$$y_3(n)\overset{?}{=}ay_1(n)+by_2(n)$$

<u>Bu eşitlik</u>
Sağlanıyor ise sistem ***lineer (doğrusal)***
Sağlanmıyor ise ***nonlineer (doğrusal değil)***

Örnek: $y(n)=2\,x(n)+3=M$ sistemin özellikleri hakkında bilgi veriniz?

$$y(n)=2\,x(n)+3=M$$

$$y(n-k)=T[x(n-k)]=2\,x(n-k)+3$$

$$eşit\left\langle \begin{array}{l} y_1(n)=2\,x_1(n)+3=2\,x(n-k)+3 \\ y(n-k)=2\,x(n-k)+3 \end{array}\right.$$

- (n-k) yok Hafızalı Değil
- x(n) var Nedensel
- $y_1(n)=y(n-k)$ eşit olduğu için; Zamanla değişmez (Sabit n yok)
- Kararlı n ∞'a gitmiyor
- $y_3(n)\neq T[x_3(n)]$ eşit olmadığı için; Doğrusal değil

$$y_1(n)=T[x_1(n)]=2x_1(n)+3$$
$$y_2(n)=T[x_2(n)]=2x_2(n)+3$$

$$x_3(n)=ax_1(n)+bx_2(n)$$
$$y_3(n)=T[x_3(n)]=2(ax_1(n)+bx_2(n))+3$$

$$T[x_3(n)]=aT[x_1(n)]+bT[x_2(n)]$$
$$T[x_3(n)]=a(2x_1(n)+3)+b(2x_2(n)+3)$$

$$y_3(n)\overset{?}{=}T[x_3(n)]$$
$$2(ax_1(n)+bx_2(n))+3\neq a(2x_1(n)+3)+b(2x_2(n)+3)$$

Örnek: $y(t)=(x(t))^2$ doğrusal mı?

$$y_1(t)=T[x_1(t)]=(x_1(t))^2$$
$$y_2(t)=T[x_2(t)]=(x_2(t))^2$$

$$x_3(t)=ax_1(t)+bx_2(t)$$
$$y_3(t)=T[x_3(t)]=(ax_1(t)+bx_2(t))^2$$

$$T[x_3(t)]=aT[x_1(t)]+bT[x_2(t)]$$
$$T[x_3(t)]=a(x_1(t))^2+b(x_2(t))^2$$
$$y_3(t)\overset{?}{=}T[x_3(t)]$$
$$(ax_1(t)+bx_2(t))^2\overset{?}{=}a(x_1(t))^2+b(x_2(t))^2$$
$$a^2y_1(t)+b^2y_2(t)+2aby_1(t)y_2(t)\neq ay_1(t)+by_2(t)$$

Örnek: $y(n)=T[x(n)]=n^2x(n+2)$ doğrusal mı?

$$y_1(n)=T[x_1(n)]=n^2x_1(n+2)$$
$$y_2(n)=T[x_2(n)]=n^2x_2(n+2)$$

$$x_3(n+2)=ax_1(n+2)+bx_2(n+2)$$
$$y_3(n)=T[x_3(n)]=n^2(ax_1(n+2)+bx_2(n+2))$$

$$T[x_3(n)]=aT[x_1(n)]+bT[x_2(n)]$$
$$T[x_3(n)]=a(n^2x_1(n+2))+b(n^2x_2(n+2))$$
$$y_3(n)\overset{?}{=}T[x_3(n)]$$
$$n^2(ax_1(n+2)+bx_2(n+2))\overset{?}{=}a(n^2x_1(n+2))+b(n^2x_2(n+2))$$
$$ay_1(n)+by_2(n)=ay_1(n)+by_2(n)$$

- (n+k) var Hafızalı
- x(n) ve/veya x(n-k) yok Nedensel değil
- $y_1(n)=y(n-k)$ eşit olduğu için; Zamanla değişmez değil (Sabit n var)
- Kararlı değil n ∞'a gidiyor
- $y_3(n)=T[x_3(n)]$ eşit olduğu için; Doğrusal

Örnek: $y(n) = 6\,x^2(n-3)$ sistemin özellikleri hakkında bilgi veriniz?

- (n-k) var Hafızalı
- x(n-k) var Nedensel
- $y_1(n) = y(n-k)$ eşit olduğu için;
  Zamanla değişmez (Sabit n yok)
- Kararlı n ∞'a gitmiyor
- $y_3(n) \neq T[x_3(n)]$ eşit olmadığı için;
  Doğrusal değil

$$y(n) = 6\,x^2(n-3)$$

$$x_1(n) = x(n-k)$$
$$y(n-k) = T[x(n-k)] = 6\,x^2(n-k-3)$$

$$eşit \left\langle \begin{array}{l} y_1(n) = 6\,x^2(n-k-3) \\ y(n-k) = 6\,x^2(n-k-3) \end{array} \right.$$

$$y_1(n) = T[x_1(n)] = 6\,{x_1}^2(n-3)$$
$$y_2(n) = T[x_2(n)] = 6\,{x_2}^2(n-3)$$

$$x_3(n) = ax_1(n) + bx_2(n)$$
$$y_3(n) = T[x_3(n)] = 6(ax_1(n-3) + bx_2(n-3))^2$$

$$T[x_3(n)] = aT[x_1(n)] + bT[x_2(n)]$$
$$T[x_3(n)] = a6({x_1}^2(n-3)) + b6({x_2}^2(n-3))$$

$$y_3(n) \overset{?}{=} T[x_3(n)]$$
$$6(ax_1(n-3) + bx_2(n-3))^2 \overset{?}{=} a6({x_1}^2(n-3)) + b6({x_2}^2(n-3))$$
$$6(ax_1(n-3) + bx_2(n-3))^2 \neq ay_1(n) + by_2(n)$$

Örnek: $y(t) = x(-t+1)$ sistemin özellikleri hakkında bilgi veriniz?

DOĞRUSAL ZAMANLA DEĞİŞMEZ SİSTEM (DZD)

$$y(n) = T[x(n)]$$

$$h(n) = T[\delta(n)]$$

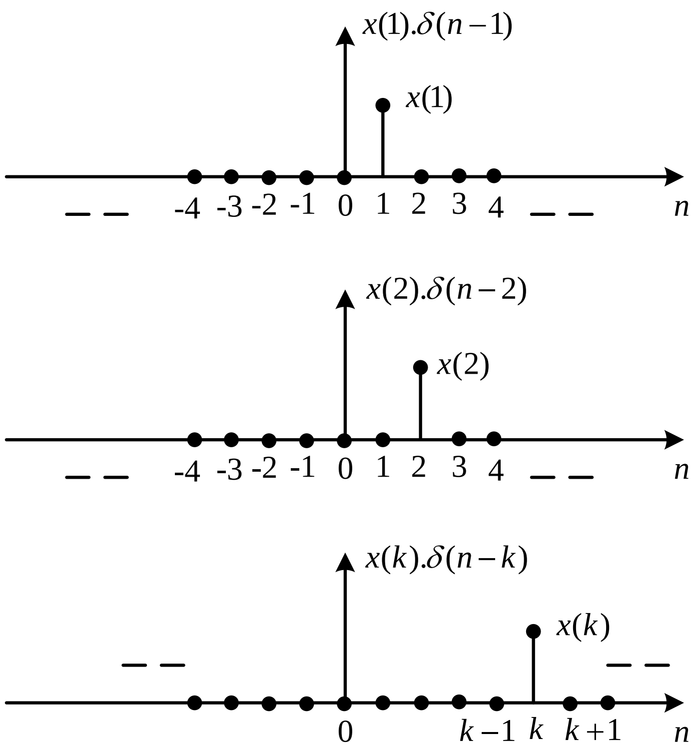
Giriş birim impuls dizisi $\delta(n)$ ise, buna karşı düşen sistem çıkışı impuls cevabı olarak adlandırılır ve $h(n)$ ile gösterilir. Ayrık zamanlı DZD sistemin giriş ve çıkış bağıntısını, birim impuls cevabı yardımıyla belirlemede ilk adım, önceki bölümde verilmiş olan denklemin bir daha yazılmasıdır.

$$x(n) = \sum_{k=-\infty}^{\infty} x(k).\delta(n-k)$$

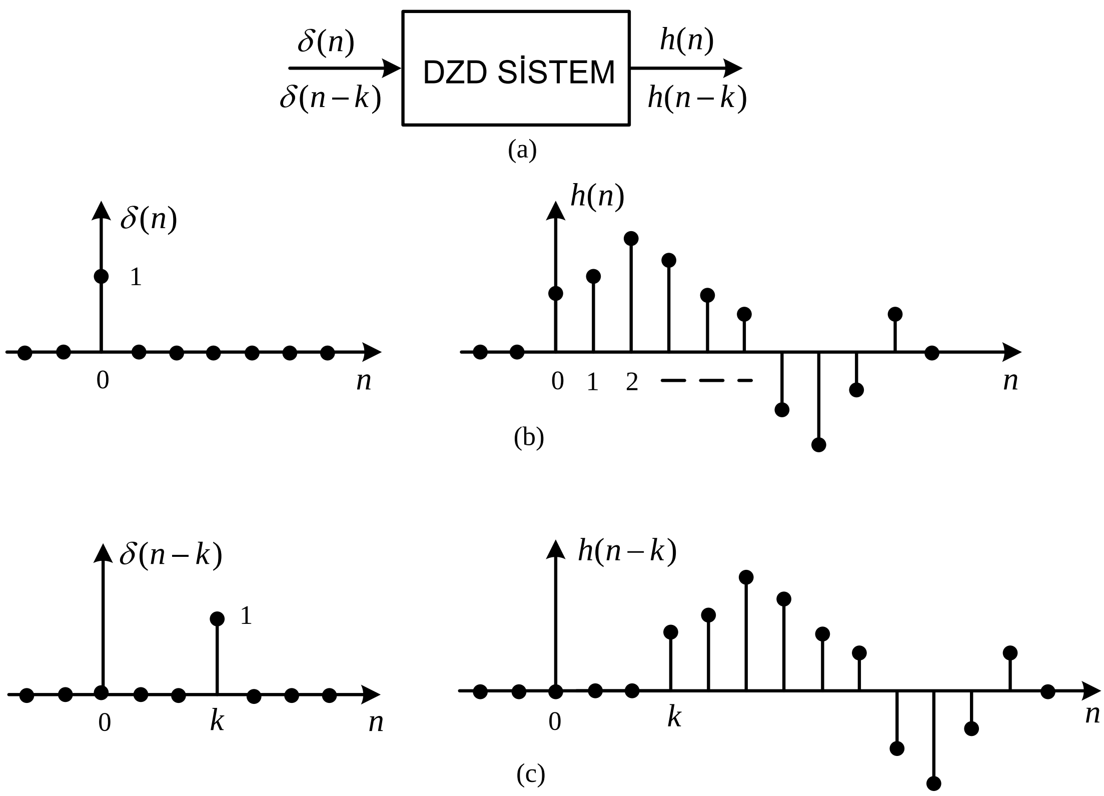
$$y(n) = T\big[x(n)\big] = T\left[\sum_{k=-\infty}^{\infty} x(k).\delta(n-k)\right] = \sum_{k=-\infty}^{\infty} x(k).T\big[\delta(n-k)\big] = \sum_{k=-\infty}^{\infty} x(k).h(n-k)$$

Konvalisyon Toplamı

$$y(n) = \sum_{k=-\infty}^{\infty} x(k).h(n-k)$$



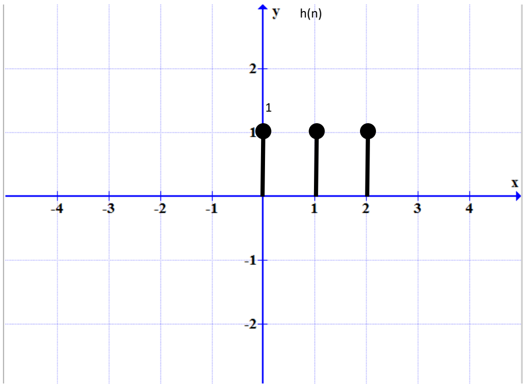
$x(n)$ dizisinin impuls bileşenleri ile gösterilmesi

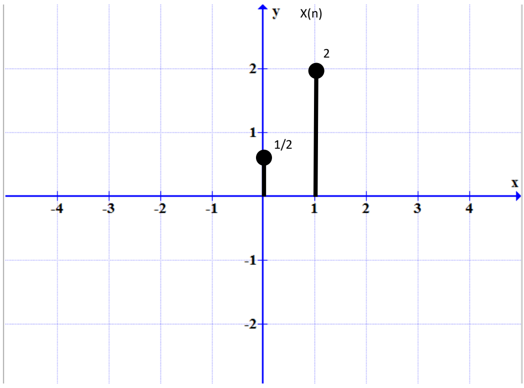


Zamanla değişmeme kriteri, (a) DZD sistemin ötelenmiş birim impuls cevabı, (b) $\delta(n)$ için DZD sistemin cevabı, (c) $\delta(n-k)$ için aynı sistemin cevabı

Örnek $x(n)$, birim impuls cevabı $h(n)$ olarak verilen DZD bir sisteme giriş olarak uygulanmaktadır.

Çıkış dizisini bulunuz? $y(n)=\sum_{k=-\infty}^{\infty} x(k)h(n-k)$

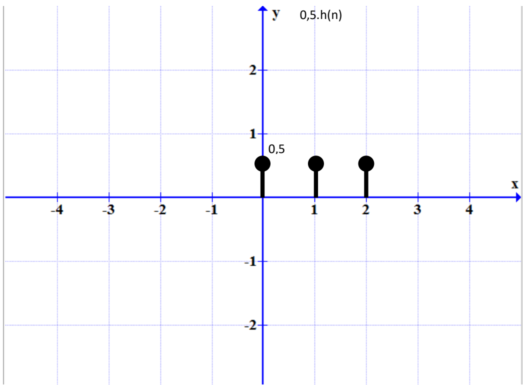


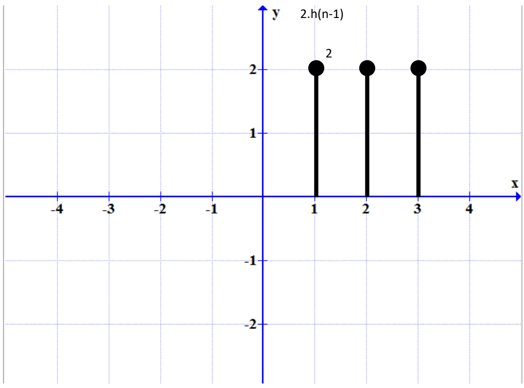


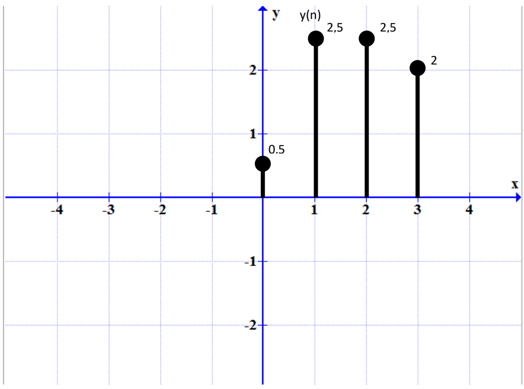
Çözüm

$$y(n)=\sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n)=x[0]h[n-0]+x[1]h[n-1]=0,5h(n)+2h(n-1)$$





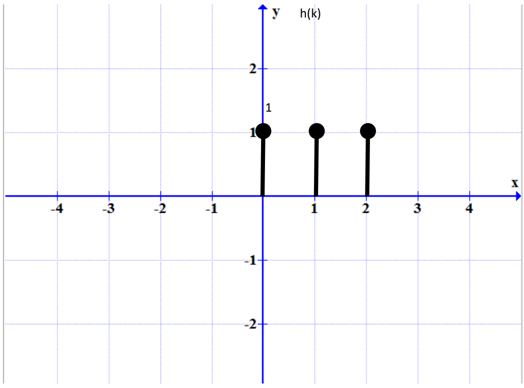


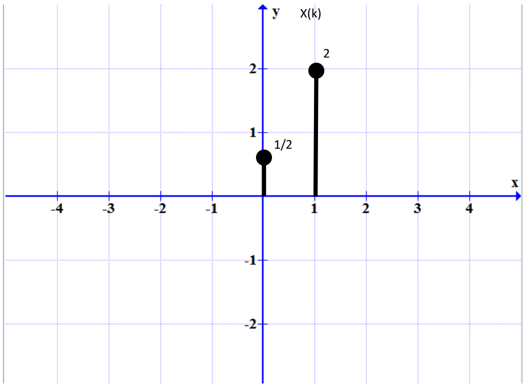
∞'a gidiyorsa;

$$y(n)=\sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

k'lı ifadeye dönüştürüp topluyoruz

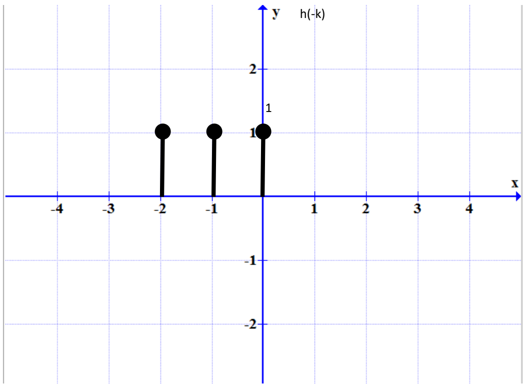
Örnek $x(n)$, $h(n)$ olarak verilen DZD bir sisteme giriş olarak uygulanmaktadır. Çıkış dizisini bulunuz? $y(n) = ?$

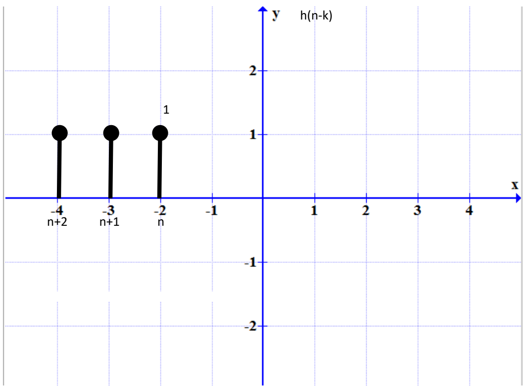




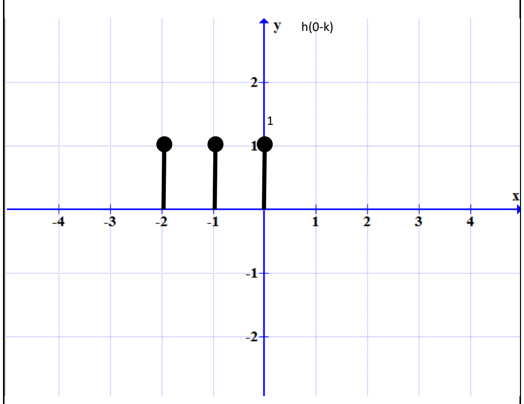
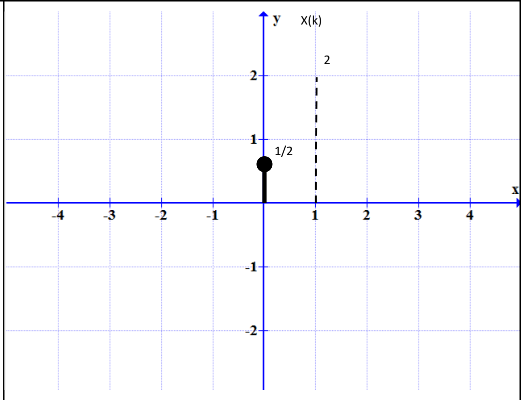
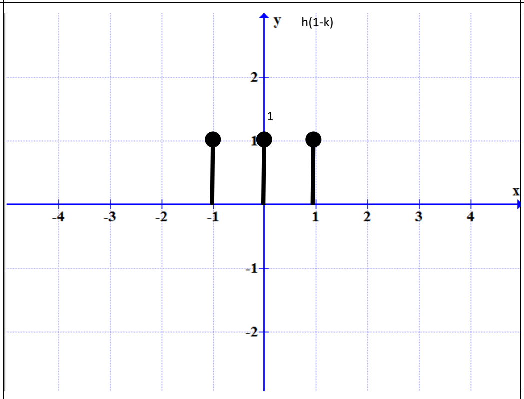
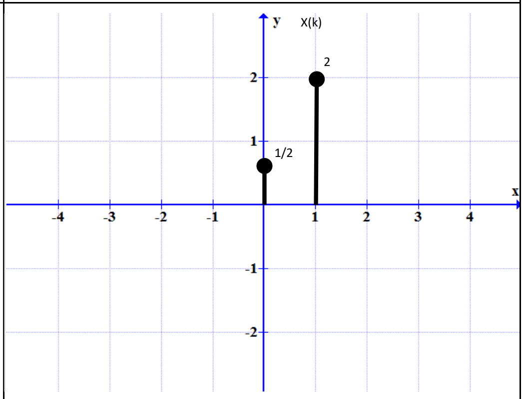
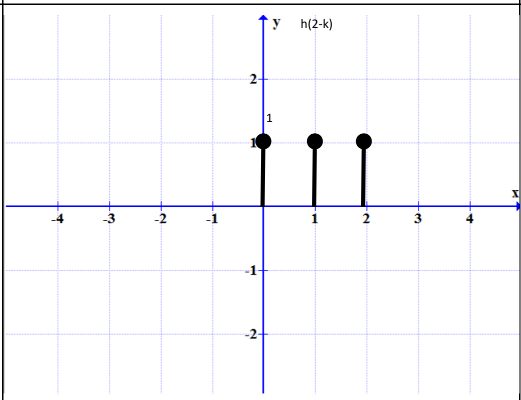
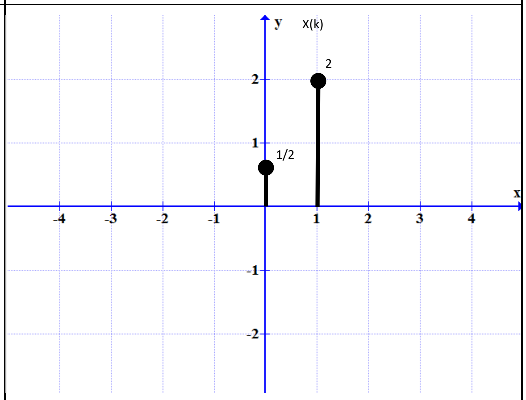
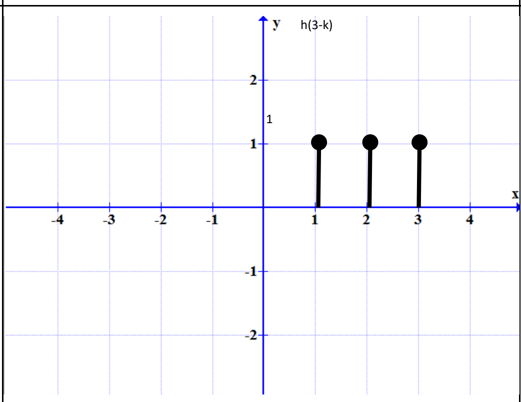
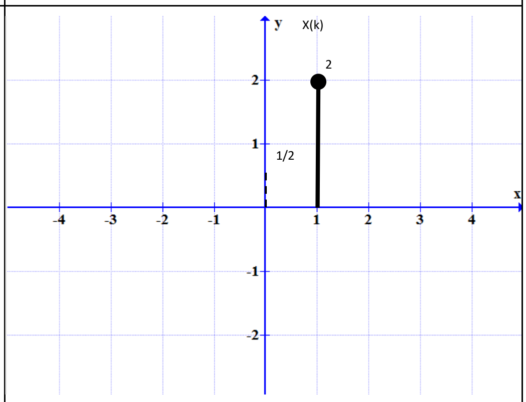
Çözüm

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$





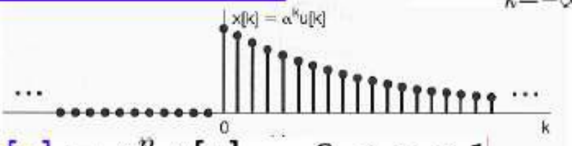
$n < 0$	Hiç örtüşme yok $y(n) = 0$
$n = 0$	$y(0) = 0,5 + 0 = 0,5$
$n = 1$	$y(1) = 0,5 + 2 = 2,5$
$n = 2$	$y(2) = 0,5 + 2 = 2,5$
$n = 3$	$y(3) = 0 + 2 = 2$
$n \geq 4$	Hiç örtüşme yok $y(n) = 0$

$n<0$	Hiç örtüşme yok	$y(n)=0$	
$n=0$		$y(0)=0,5$ → *çarp*	
$n=1$		$y(1)=0,5+2$ → *çarp*	
$n=2$		$y(2)=0,5+2$ → *çarp*	
$n=3$		$y(3)=0+2$ → *çarp*	
$n\geq 4$	Hiç örtüşme yok	$y(n)=0$	

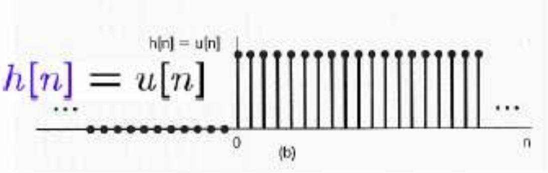
Örnek Chapter2.pdf/Example2.3

- Example 2.3:

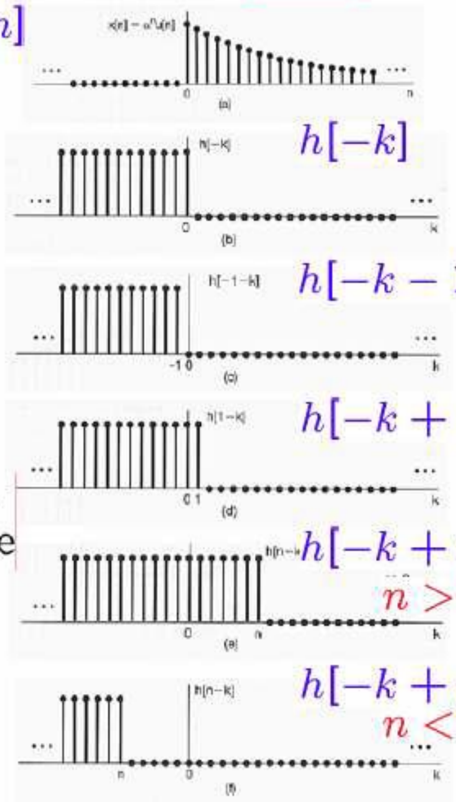
$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \qquad x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$



$x[n] = \alpha^n \, u[n], \quad 0 < \alpha < 1$



$h[n] = u[n]$



for $n < 0, \quad x[k]\, h[n-k] = 0 \ \Rightarrow \ y[n] = 0$

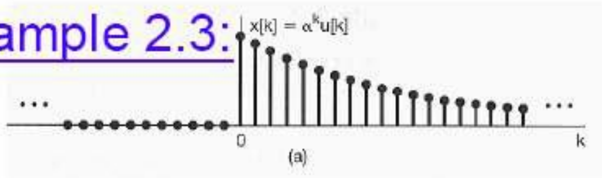
for $n \geq 0, \quad x[k]\, h[n-k] = \begin{cases} \alpha^k, & 0 \leq k \leq n \\ 0, & \text{otherwise} \end{cases}$

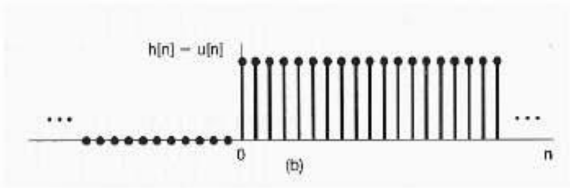
$$\Rightarrow y[n] = \sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}$$

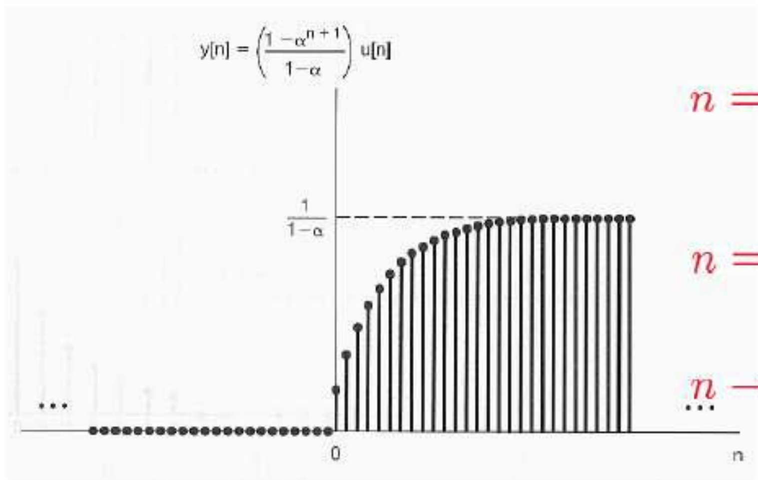
for all $n, \quad y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right) u[n]$

- Example 2.3:

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$







$$\alpha = \frac{7}{8}$$

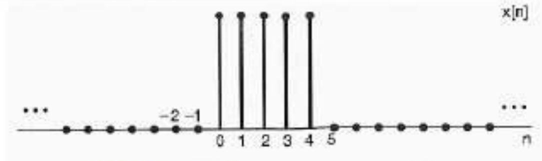
$$n = 0 \quad y[0] = \frac{1-\frac{7}{8}}{1-\frac{7}{8}} = 1$$

$$n = 1 \quad y[1] = \frac{1-(\frac{7}{8})^2}{1-\frac{7}{8}} = \frac{15}{8}$$

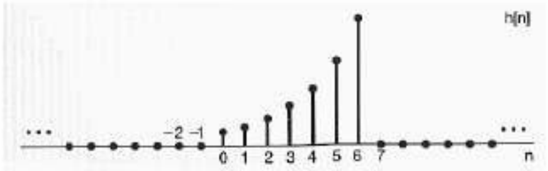
$$n \to \infty \quad y[n] = \frac{1-0}{1-\frac{7}{8}} = 8$$

Örnek Chapter2.pdf/Example 2.4

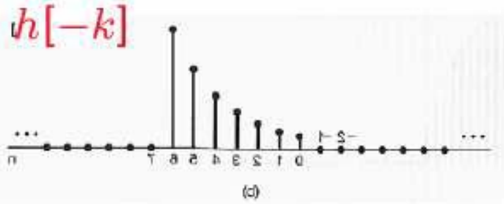
Example 2.4:



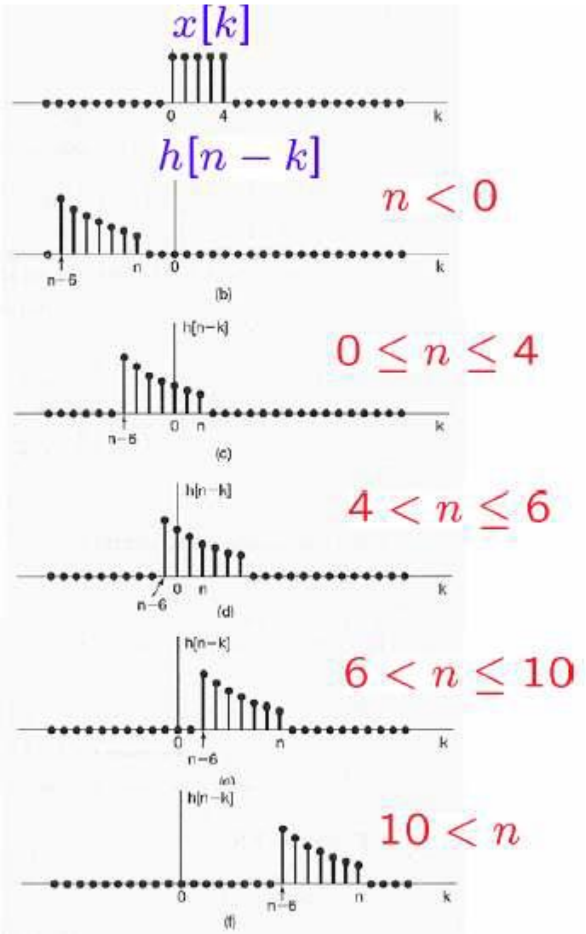
$x[n] = 1, \; 0 \le n \le 4$

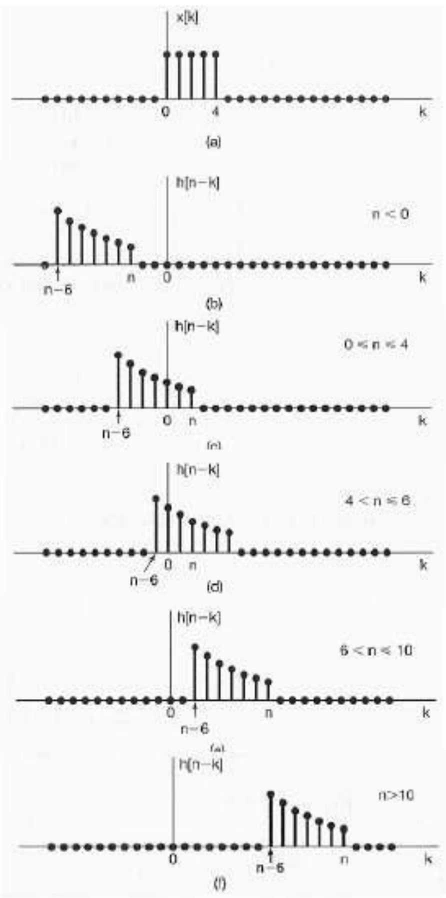


$h[n] = \alpha^n, \; 0 \le n \le 6$



$x[n] \longrightarrow h[n] \longrightarrow y[n]$





$x[n] \longrightarrow h[n] \longrightarrow y[n]$

for $n < 0$, $\quad x[k]\,h[n-k] = 0 \;\Rightarrow\; y[n] = 0$

for $0 \le n \le 4$, $\quad x[k]\,h[n-k] = \begin{cases} \alpha^{n-k}, & 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}$

$$\Rightarrow y[n] = \sum_{k=0}^{n} \alpha^{n-k} = \frac{1-\alpha^{n+1}}{1-\alpha}$$

for $4 < n \le 6$, $\quad x[k]\,h[n-k] = \begin{cases} \alpha^{n-k}, & 0 \le k \le 4 \\ 0, & \text{otherwise} \end{cases}$

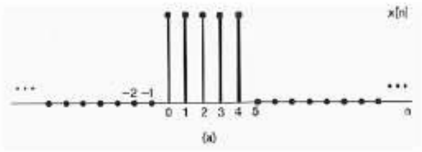
$$\Rightarrow y[n] = \sum_{k=0}^{4} \alpha^{n-k} = \frac{\alpha^{n-4}-\alpha^{n+1}}{1-\alpha}$$

for $6 < n \le 10$, $\quad x[k]\,h[n-k] = \begin{cases} \alpha^{n-k}, & (n-6) \le k \le 4 \\ 0, & \text{otherwise} \end{cases}$

$$\Rightarrow y[n] = \sum_{k=n-6}^{4} \alpha^{n-k} = \frac{\alpha^{n-4}-\alpha^{7}}{1-\alpha}$$

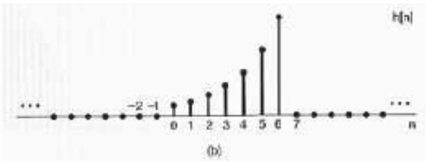
for $n > 10$, $\quad y[n] = 0$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$



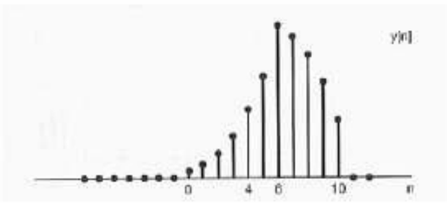
(a)

$$x[n] = 1, \; 0 \le n \le 4$$



(b)

$$h[n] = \alpha^n, \; 0 \le n \le 6$$

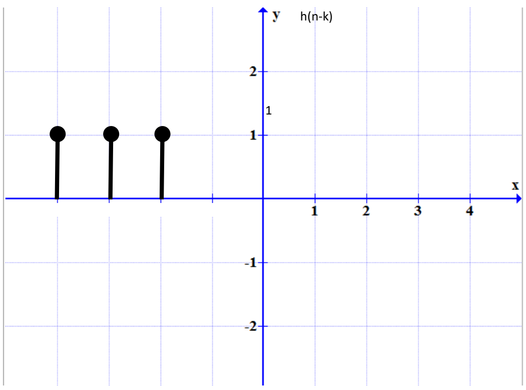


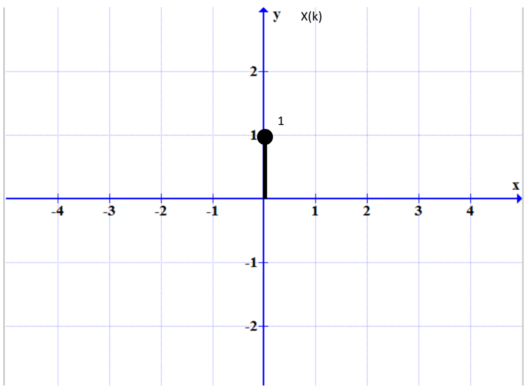
$$y[n] = \begin{cases} 0, & n < 0 \\ \frac{1-\alpha^{n+1}}{1-\alpha}, & 0 \le n \le 4 \\ \frac{\alpha^{n-4}-\alpha^{n+1}}{1-\alpha}, & 4 < n \le 6 \\ \frac{\alpha^{n-4}-\alpha^{7}}{1-\alpha}, & 6 < n \le 10 \\ 0, & 10 < n \end{cases}$$

Örnek $\sum x(k).h(n-k) = x(n) * h(n)$

$$x(n) \rightarrow \boxed{h(n)} \rightarrow x(n) * h(n)$$

$$x(n) = \delta(n)$$
$$y(n) = ?$$





Çözüm

$$x(n) = \delta(n)$$
$$y(n) = x(n) * h(n) = \delta(n) * h(n) = h(n)$$

$$y(n) = u(n)$$

$n < 0$	$y(n) = 0$
$n = 0$	$y(0) = 1$
$n = 1$	$y(1) = 1$
$n = 2$	$y(2) = 1$
$\vdots$	
$n = \infty$	$y(n) = 1$

Örnek $\sum x(k).h(n-k) = x(n) * h(n)$

$$x(n) \rightarrow \boxed{h(n)} \rightarrow x(n) * h(n)$$

$$x(n) = \delta(n-1)$$
$$y(n) = ?$$

Konvalisyon:

$$y(n) = \delta(n-1) * h(n) = h(n-1)$$
$$y(n) = \delta(n-2) * x(n) = x(n-2)$$

$$y(n) = \delta(n+1) * x(n) = x(n+1)$$
$$y(n) = \delta(n+k) * x(n) = x(n+k)$$

Çözüm

$$x(n) = \delta(n-1)$$
$$y(n) = x(n) * h(n) = \delta(n-1) * h(n)$$

$$y(n) = u(n-1) = h(n-1)$$

Birim gecikme elemanı;

$$x(n) \rightarrow \boxed{h(n)} \rightarrow x(n-1)$$

Birim gecikme elemanı

$$z^{-1}$$

$$z^{-1}$$
$$x(n-1) = x(n) * h(n)$$
$$x(n-1) = x(n) * \delta(n-1)$$
$$x(n-1) = x(n) * z^{-1}$$

Örnek

$$x(n) = \delta(n) + \delta(n-2)$$
$$h(n) = \delta(n-2) + \delta(n+2)$$
$$y(n) = ?$$

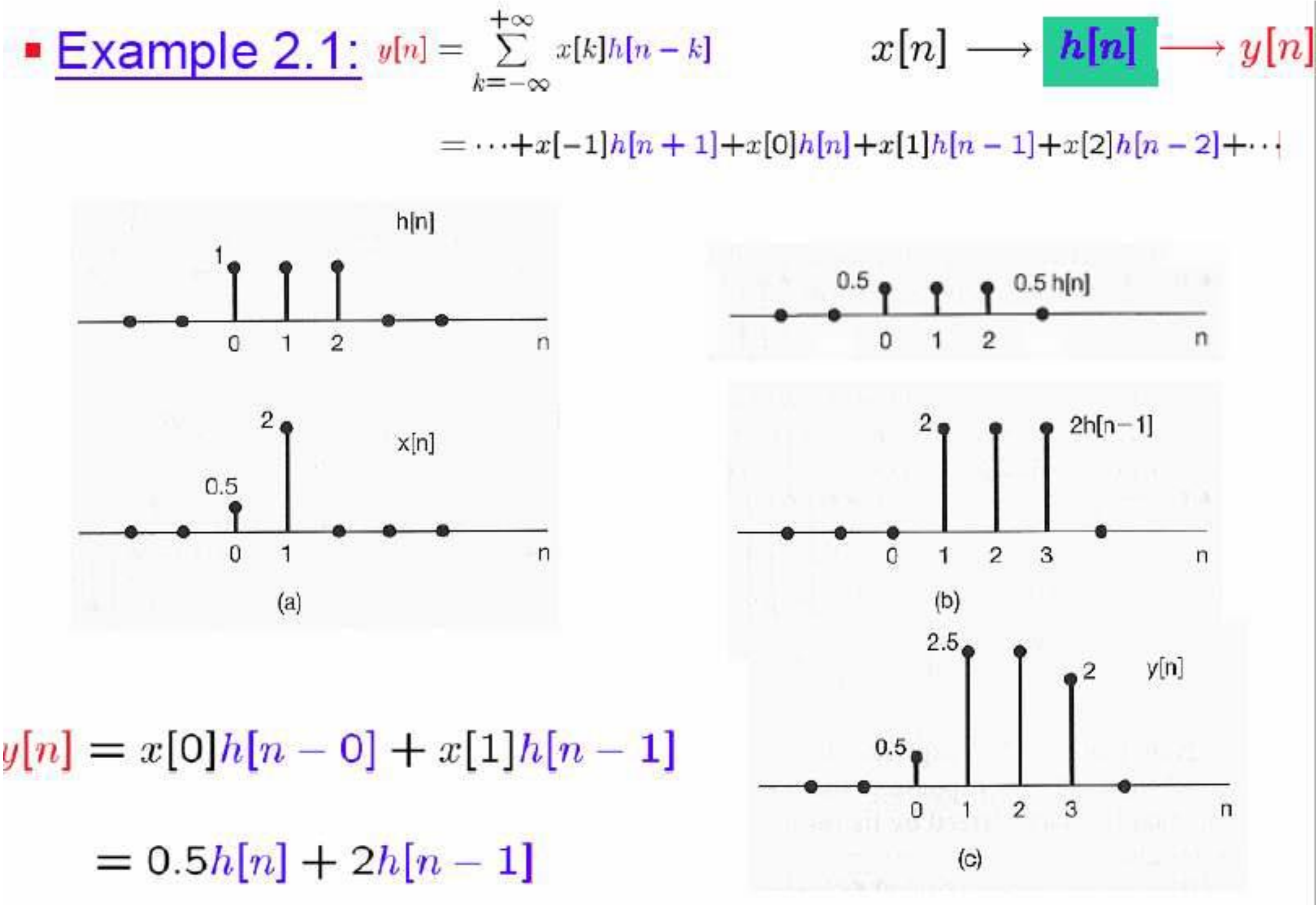
Çözüm

$$y(n) = x(n) * h(n)$$
$$y(n) = h(n) * x(n)$$

$$y(n) = x(n) * h(n)$$
$$= [\delta(n) + \delta(n-2)] * h(n)$$
$$= \underbrace{\delta(n) * h(n)}_{h(n)} + \underbrace{\delta(n-2) * h(n)}_{h(n-2)}$$
$$= \delta(n-2) + \delta(n+2) + \delta((n-2)-2) + \delta((n-2)+2)$$
$$= \delta(n-2) + \delta(n+2) + \delta(n-4) + \delta(n)$$

Örnek Chapter2.pdf/Example2.1

$$h(n)=\delta(n)+\delta(n-1)+\delta(n-2)$$
$$x(n)=0,5\,\delta(n)+2\,\delta(n-1)$$
$$y(n)=?$$



$$h(n)=\delta(n)+\delta(n-1)+\delta(n-2)$$
$$x(n)=0,5\,\delta(n)+2\,\delta(n-1)$$

$$\begin{aligned}
y(n)&=x(n)*h(n)\\
&=x(n)*[\delta(n)+\delta(n-1)+\delta(n-2)]\\
&=\underbrace{x(n)*\delta(n)}_{x(n)}+\underbrace{x(n)*\delta(n-1)}_{x(n-1)}+\underbrace{x(n)*\delta(n-2)}_{x(n-2)}\\
&=x(n)+x(n-1)+x(n-2)\\
&=0,5\,\delta(n)+2\,\delta(n-1)+0,5\,\delta(n-1)+2\,\delta(n-2)+0,5\,\delta(n-2)+2\,\delta(n-3)\\
&=0,5\,\delta(n)+2,5\,\delta(n-1)+2,5\,\delta(n-2)+2\,\delta(n-3)
\end{aligned}$$

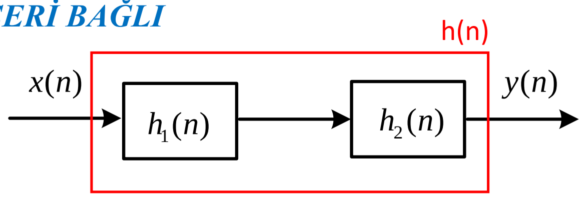
Ödev

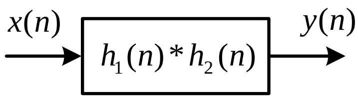
$h(n)=\alpha^n \quad 0\le n\le 4$

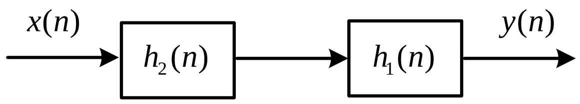
$x(n)=1 \quad 0\le n\le 6$

olduğunda $y(n)=?$

SERİ BAĞLI







$$y(n) = x(n) * h(n) = \sum x(k).h(n-k)$$

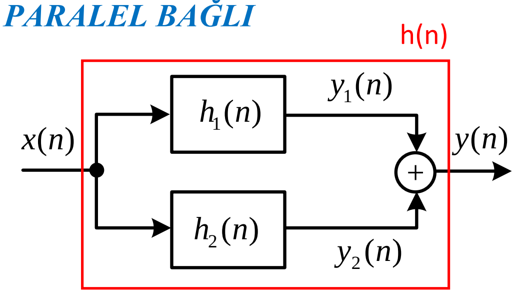
$$= x(t) * h(t) = \sum x(\tau).h(t-\tau)d\tau$$

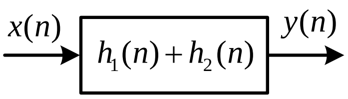
$$= x(n) * h(n)$$

$$y_1(n) = x(n) * h_1(n)$$

$$y(n) = y_1(n) * h_2(n) = x(n) * h_1(n) * h_2(n)$$

PARALEL BAĞLI



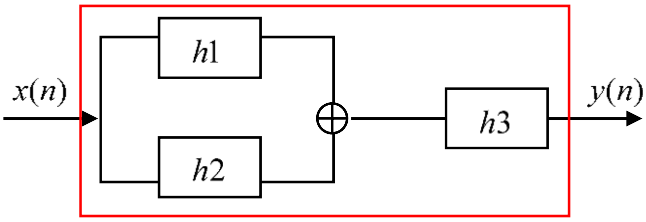


$$y(n) = x(n) * h_1(n) + x(n) * h_2(n)$$

$$= x(n) * (h_1 + h_2)$$

Çıkışta yapılan işleme göre burası değişir. Bu örnek te +

Örnek



$$h(n)=(h1+h2)*h3$$

$$\begin{cases} h_1(n)=\delta(n-1) \\ h_2(n)=\delta(n-2)+\delta(n) \\ h_3(n)=\delta(n+1) \end{cases}$$

$$h(n)=?$$

Çözüm

$$\begin{aligned} h(n) &= (h_1+h_2)*h_3(n) \\ &= (h_1+h_2)*\delta(n+1) \\ &= h_1\ \delta(n+1)+h_2\ \delta(n+1) \\ &= h_1(n+1)+h_2(n+1) \\ &= \delta(n)+\delta(n-1)+\delta(n+1) \end{aligned}$$

 # Sistemin impuls cevabına bakarak <u>hafızalı olup olmadığını</u> nasıl çözeriz

$$\begin{aligned} y(n) &= T[x(n)] \\ &= x(n)*h(n) \\ &= h(n)*x(n) \\ &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \end{aligned}$$

$$\ldots+(h-1)x(n+1)+h(0)x(n)+h(1)x(n-1)+\ldots$$

Hafızalı olma şartı;

$$h(n)=0 \qquad n\neq 0$$

$$h(n)=\delta(n-1)$$

Sistemin impuls cevabına bakarak <u>nedensel olup olmadığını</u> nasıl çözeriz

Nedensel olma şartı;

$$h(n)=0 \qquad n<0$$

$h(n)=\delta(n-1)$ Nedensel

$h(n)=3\delta(n-1)$ Nedensel

Sistemin impuls cevabına bakarak <u>Kararlı olup olmadığını</u> nasıl çözeriz

Tanım uyarınca her sınırlı giriş işareti yine sınırlı bir çıkış sağlıyorsa, DZD sistem kararlıdır.

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Örnek

DZD sistemin impuls cevabı aşağıdaki şekilde verilmiş olsun.

$n<0$ şartını $u(n)$ sağlıyor

$h(n)=a^n u(n)$ nedenseldir

$h(n)=a^n u(n)$ hafızalıdır

$$h(n)=a^n u(n)$$

$\sum_{k=-\infty}^{\infty}|h(k)|=\sum_{k=0}^{\infty}|a|^k$ elde edilir. Eğer $|a|<1$ ise toplamı yakınsar. Buradan aşağıdaki sonuca gelinir.

$$\sum_{k=0}^{\infty}|h(k)|=\frac{1}{1-|a|}$$

O halde sistem kararlıdır. Ancak $|a|\geq 1$ olursa bu toplam yakınsamaz ve sistem kararsız olur.

Örnek

DZD sistemin impuls cevabı aşağıdaki şekilde verilmiş olsun.

$$h(n)=\delta(n-n_0)$$

$h(n)=\delta(n-n_0)$ kararlı $\quad \sum_{n=-\infty}^{\infty}|h(n)|=\sum_{n=-\infty}^{\infty}|\delta(n-n_0)|=1$

$h(n)=u(n)$ kararsız $\quad \sum_{n=-\infty}^{\infty}|h(n)|=\sum_{n=-\infty}^{\infty}|u(n)|=\sum_{n=0}^{\infty}|u(n)|\rightarrow\infty$

$$h(n)=\delta(n-n_0)\begin{cases} n=0 & \textit{Hafızasız} \\ n\neq 0 & \textit{Hafızalı} \end{cases}$$

$$h(n)=\delta(n-n_0)\begin{cases} n_0<0 & \textit{Nedensel değil} \\ n_0>0 & \textit{Nedensel} \end{cases}$$

BLOK DİYAGRAMLAR

Fark Denklemleri

$h(n)$ ***FIR*** *sınırlı sayıda toplama, çıkarma*

IIR *sonsuz sayıda* //

$$\sum_{k=0}^{N} b_k y(n-k) = \sum_{k=0}^{M} a_k x(n-k) \qquad a_k, b_k \text{ sabit sayılar}$$

$$\delta(n-1) = z^{-1}$$

Sistemin Cevabının hesaplanması

$$\sum b_k y(n-k) = \sum a_k x(n-k)$$ başlangıç değerleri veriliyor

$$\left.\begin{matrix} y(-1) \\ y(-2) \end{matrix}\right\}$$ 2.dereceden denklem ise 2 giriş işareti verilir.

$x(n)$

$y(n) = ?$

Doğal Çözüm

Girişi işareti $x(n) = 0$ kabul edilir

$y(-1)$ *ve* $y(-2)$ verilenler kullanılarak sistemin doğal çözümü $y_d(n)$ bulunur

$y(n) = \lambda^n$ fark denkleminde yerine yazılır, kökler bulunur

Zorlanmış Çözüm

Doğal çözümün tam tersi

Başlangıç koşulları $y(-1) = 0$ *ve* $y(-2) = 0$ kabul edilir.

Verilen giriş işaretine $x(n)$ göre (***bkz sf 30*** Tablo) sistemin zorlanmış çözümü $y_z(n)$ bulunur

Özel Çözüm

$y(n)=\lambda^n$ kabul edilir. Doğal çözümde fark denklemine yazılır

n tane kök bulunur; $\lambda_1, \lambda_2, \lambda_3 \cdots$

$$y_d(n)=c_1\lambda_1^n+c_2\lambda_2^n+\quad\cdots\quad+c_N\lambda_N^n$$

Başlangıç koşulları 0 kabul ediliyor. Başlangıç koşullarıyla c leri buluyoruz

$$y(n)=\sum x(n-k)+\sum b_k y(n-k) \qquad \underbrace{y(0), y(1)}_{\text{2.dereceden ise ikisi kullanılıyor}}, y(2)$$

$$y(n)+b_1y(n-1)+b_2y(n-2)=\sum x(n-k)$$

$$\lambda^n+b_1\lambda^{n-1}+b_2\lambda^{n-2}=0$$

$$\lambda^{n-2}\underbrace{(\lambda^2+b_1\lambda^n+b_2)}_{\lambda_1 \text{ ve } \lambda_2 \text{ kökler}}=0$$

$\lambda_1 \neq \lambda_2 \qquad c_1\lambda_1^n+c_2\lambda_2^n=y_d(n)$

$\lambda_1 = \lambda_2 \qquad c_1\lambda_1^n+c_2n\lambda_1^n=y_d(n)$

$\lambda_1 = \lambda_2 = \lambda_3 \qquad c_1\lambda_1^n+c_2n\lambda_1^n+c_3n^2\lambda_1^n=y_d(n)$

Katlı kök varsa

$$0 \rightarrow y(0)+b_1y(0-1)+b_2y(0-2)=y(0)+b_1y(-1)+b_2y(-2)=0 \rightarrow y(0)$$
$$1 \rightarrow y(1)+b_1y(1-1)+b_2y(1-2)=y(1)+b_1y(0)+b_2y(-1)=0 \rightarrow y(1)$$

c_1 ve c_2 bulunur

$x(n)$	$y_{Ö}(n)$
$Au(n)$	$Ku(n)$
$AM^nu(n)$	$KM^nu(n)$
An^m	$K_0n^m+K_1n^{m-1}+\cdots+K_m$
$A\cos(u(n))$ $A\sin(u(n))$	$K_0\cos(\omega_0 n)+K_1\sin(\omega_0 n)$

$$y_z(n)=y_d(n)+y_{Ö}(n)$$

$$Toplam\ çözüm=y_d(n)+y_z(n)$$

Örnek

$$y(-1)=2$$
$$y(-2)=2$$
$$y(n)-2y(n-1)-3y(n-2)=x(n)$$
$$y_d(n)=?$$

Çözüm

$$y(n)=\lambda^n$$

$$y(n)-2y(n-1)-3y(n-2)=x(n)$$
$$\underbrace{\lambda^n-2\lambda^{n-1}-3\lambda^{n-2}}_{\lambda_1=-1 \text{ ve } \lambda_2=3}=0$$

$$y_d(n)=c_1\lambda_1^n+c_2\lambda_2^n$$
$$=c_1(-1)^n+c_2(3)^n$$
$$=1(-1)^n+9(3)^n$$
$$=(-1)^n+(3)^{n+2}$$

$$y_d(n)=c_1\lambda_1^n+c_2\lambda_2^n=c_1(-1)^n+c_2(3)^n$$

$y(0)$ *ve* $y(1)$ kullanılarak c_1 ve c_2 bulunur

$$y(n)-2y(n-1)-3y(n-2)=x(n)$$

$$0\rightarrow y(0)-2y(-1)-3y(-2)=0$$
$$1\rightarrow y(1)-2y(0)-3y(-1)=0$$

$$y(0)-2.2-3.2=0$$
$$y(0)=10=c_1+c_2$$

$$y(1)-2.10-3.2=0$$
$$y(1)=26=-c_1+3c_2$$

$$10=c_1+c_2$$
$$26=-c_1+3c_2$$

$$c_1=1$$
$$c_2=9$$

Örnek

$y(n)+ay(n-1)=x(n)$ doğal ve homojen çözümünü bulunuz?

Çözüm

$$y(n)=\lambda^n$$

$$y(n)+ay(n-1)=x(n)$$
$$\underbrace{\lambda^n+a\lambda^{n-1}}_{\lambda_1=-a}=0$$

$$y_d(n)=c_1\lambda_1^n$$
$$=c_1(-a)^n$$
$$=-ay(-1)(-a)^n$$
$$=(-a)^{n+1}y(-1)$$

$$y_d(n)=c_1\lambda_1^n=c_1(-a)^n$$

$y(0)$ kullanılarak c_1 bulunur

$$y(n)+ay(n-1)=x(n)$$

$$0\rightarrow y(0)+ay(-1)=0$$
$$1\rightarrow y(1)+ay(0)=0$$

$$y(0)+ay(-1)=0$$
$$y(0)=-ay(-1)=c_1$$

$$c_1=-ay(-1)$$

Örnek

$$\begin{aligned} &y(-2)=0, \quad y(-1)=5 \\ &y(n)-3y(n-1)-4y(n-2)=x(n) \end{aligned}$$ doğal çözümünü bulunuz?

Çözüm

$$y(n)=\lambda^n$$

$$y(n)-3y(n-1)-4y(n-2)=x(n)=x(n)$$

$$\underbrace{\lambda^n-3\lambda^{n-1}-4\lambda^{n-2}}_{\lambda_1=4 \quad \lambda_2=-1}=0$$

$$\begin{aligned} y_d(n) &= c_1\lambda_1^n + c_2\lambda_2^n \\ &= 16.4^n + -1.(-1)^n \\ &= 4^{n+2} + (-1)^{n+1} \end{aligned}$$

$$y_d(n)=c_1\lambda_1^n+c_2\lambda_2^n=c_1(4)^n+c_2(-1)^n$$

$y(0)$ *ve* $y(1)$ kullanılarak c_1 ve c_2 bulunur

$$y(n)-3y(n-1)-4y(n-2)=x(n)$$

$$\begin{aligned} &0 \rightarrow y(0)-3y(-1)-4y(-2)=0 \\ &1 \rightarrow y(1)-3y(0)-4y(-1)=0 \end{aligned}$$

$$\begin{aligned} &y(0)-3y(-1)-4y(-2)=0 \\ &y(0)=3.5+4.0=15=c_1+c_2 \end{aligned}$$

$$\begin{aligned} &y(1)-3y(0)-4y(-1)=0 \\ &y(1)=3.15+4.5=65=4c_1-c_2 \end{aligned}$$

$$\begin{aligned} &15=c_1+c_2 \\ &65=4c_1-c_2 \\ \hline &c_1=16 \\ &c_2=-1 \end{aligned}$$

Örnek (Özel Çözüm)

$y(-2)=0, \quad y(-1)=5$ *verilmiş fakat zorlanmış çözümde başlangıç koşulları 0 kabul ediliyor*

$$\begin{aligned} y(n)-3y(n-1)-4y(n-2)&=x(n) \\ Ku(n)-3Ku(n-1)-4Ku(n-2)&=Au(n) \\ K-3K-4K&=A \end{aligned}$$

$$\text{Tam Çözüm yada Toplam Çözüm}=y_d+y_ö$$

Örnek

$y(n)-2y(n-1)-3y(n-2)=x(n)$ fark denkleminin Özel ve Zorlanmış çözümünü bulunuz?

Çözüm

$$y(n)-2y(n-1)-3y(n-2)=x(n)$$

Doğal Çözümü daha önce hesaplamıştık (bkz.)

$$y_d(n)=(-1)^n+9(3)^n$$
$$y_z(n)=y_d(n)+y_{Ö}(n)$$
$$y_z(n)=c_3(-1)^n+c_4(3)^n+y_{Ö}(n)$$

$x(n)=10u(n)$ //Giriş İşareti

$$y_{Ö}(n)=Ku(n)$$

$$y(n)-2y(n-1)-3y(n-2)=x(n)$$

$$Ku(n)-2Ku(n-1)-3Ku\underbrace{(n-2)}_{\text{En fazla ötelenen } n>2 \text{ durumlar için}}=10u(n)$$

$$K-2K-3K=10$$
$$-4K=10$$
$$K=-\frac{5}{2}$$

$$y_z(n)=c_3(-1)^n+c_4(3)^n-\frac{5}{2}u(n)$$

Başlangıç koşulları 0

$n=0$

$$y(0)-\underbrace{\cancel{y(-1)}}_{0}-3\underbrace{\cancel{y(-2)}}_{0}=x(0)$$
$$y(0)=x(0)$$
$$y(0)=10$$
$$10=c_3+c_4-\frac{5}{2}$$

$n=0$

$$y(1)-\underbrace{y(0)}_{10}-3\underbrace{\cancel{y(-1)}}_{0}=x(1)$$
$$y(1)-10=x(1)$$
$$y(1)=x(1)+10$$
$$y(1)=20+10$$
$$y(1)=30$$
$$30=-c_3+3c_4-\frac{5}{2}$$

$$10=c_3+c_4-\frac{5}{2}$$
$$30=-c_3+3c_4-\frac{5}{2}$$
$$c_3=0,875$$
$$c_4=11,125$$

$$y_z(n)=0,875\,(-1)^n+11,125\,(3)^n-2,5\,u(n)$$

Örnek

$y(n)=\frac{5}{6}y(n-1)-\frac{1}{6}y(n-2)+x(n)$ $\quad n\geq 0 \quad x(n)=2^n u(n) \quad y_{Ö}(n)=?$

Çözüm

$y(n)=\frac{5}{6}y(n-1)-\frac{1}{6}y(n-2)+x(n)$ $\quad x(n)$ yalnız bırak

$y_{Ö}(n)=K2^n u(n)$

$$y(n)-\frac{5}{6}y(n-1)+\frac{1}{6}y(n-2)=x(n)$$

$$K2^n u(n)-\frac{5}{6}K2^n u(n-1)+\frac{1}{6}K2^n u(n-2)=2^n \not{u(n)}$$

$$K2^n-\frac{5}{6}K2^{n-1}+\frac{1}{6}K2^{n-2}=2^n$$

$$2^{n-2}(K2^2-\frac{5}{6}K2^1+\frac{1}{6}K)=2^n$$

$$K2^2-\frac{5}{6}K2^1+\frac{1}{6}K=4$$

$K=\frac{8}{5}$

Örnek

$y(n)+ay(n-1)=x(n)$ $\quad n\geq 0 \quad x(n)=u(n) \quad y_d(n)=? \quad y_{Ö}(n)=? \quad y_z(n)=?$

Çözüm

$y(n)+ay(n-1)=x(n)$

Tek kök olduğu için;

$y_d(n)=c_1\lambda^n$

$\lambda^n+a\lambda^{n-1}=0$

$\lambda^{n-1}(\lambda+a)=0$

$\lambda_1=-a$

$y_d(n)=c_1(-a)^n$

$y(0)+ay(-1)=x(0)$

$y(0)+ay(-1)=0$

$y(0)=-ay(-1)$

$-ay(-1)=c_1$

$y_d(n)=-ay(-1)(-a)^n$

$x(n)=u(n)$

$y_{Ö}(n)=Ku(n)$

$Ku(n)+aKu(n-1)=u(n)$

$K+aK=1$

$K=\frac{1}{1+a}$

$y_z(n)=c_2\lambda_1+y_{Ö}(n)$

$=c_2(-a)^n+\frac{1}{1+a}u(n)$

n=0 $\quad y(0)+a\not{y(-1)}=x(0)$

$y_z(0)=c_2(-a)^0+\frac{1}{1+a}u(0)$

$y_z(n)=\frac{a}{a+1}(-a)^n+\frac{1}{1+a}u(n)$

$y(0)=x(0)$
$x(0)=u(n)$
$x(0)=u(0)$
$x(0)=1$

$c2=\frac{a}{a+1}$

Toplam Çözüm$=y_d(n)+y_z(n)$

$$=-ay(-1)(-a)^n+\frac{a}{a+1}(-a)^n+\frac{1}{1+a}u(n)=[\frac{a}{a+1}-ay(-1)](-a)^n+\frac{1}{1+a}u(n)$$

Örnek

$$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$$

$y(-1)=y(-2)=0 \qquad x(n)=4^n u(n)$ toplam çözümü bulunuz?

Çözüm

$$y(n)=\lambda^n \qquad \lambda^n-3\lambda^{n-1}-4\lambda^{n-2}=0$$

$$\lambda^{n-2}(\underbrace{\lambda^2-3\lambda-4}_{\lambda_1=-1 \quad \lambda_2=4})=0$$

$$y_d(n)=c_1\lambda_1^n+c_2\lambda_2^n$$
$$=c_1(-1)^n+c_2(4)^n$$

$$y_d(0)\Rightarrow c_1+c_2=0$$
$$y_d(1)\Rightarrow -c_1+4c_2=0$$
$$c_1=0 \qquad c_2=0$$
$$y_d(0)=0$$

Başlangıç koşulları 0 olduğu için $y_d(n)=0$ olur

n=0 $\quad y(0)-3y(-1)-4y(-2)=0 \qquad y(0)=0$

n=1 $\quad y(1)-3y(0)-4y(-1)=0 \qquad y(1)=0$

$$y_z(n)=c_3(-1)^n+c_4 4^n+y_{Ö}(n)$$

$$y_{Ö}(n)=K4^n u(n)$$

Bu şekilde bir durumla karşılaşırsak katlı kök olduğunu anlıyoruz

$y_{Ö}(n)=K4^n u(n)$ yerine

$y_{Ö}(n)=Kn4^n u(n)$ kullanıyoruz

λ_1,λ_2 kökler için $\quad \cancel{c_1\lambda_1^n+c_2\lambda_2^n}$

$$c_1\lambda_1^n+c_2 n\lambda_2^n$$

$y_{Ö}(n)=Kn4^n u(n)$ Fark denkleminde yerine yazıyoruz

$$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$$

$$Kn4^n u(n)-K(n-1)4^{n-1}u(n)-4K(n-2)\underbrace{4^{n-2}}_{ortak\ parantez}u(n)=4^n u(n)+2.\underbrace{4^{n-1}}_{ortak\ parantez}u(n-1)$$

$$\cancel{4^{n-2}}(16Kn-12K(n-1)-4K(n-2))=\cancel{4^{n-1}}^{\,4}(4+2)$$
$$16Kn-12Kn-12K-4Kn-8K)=24$$
$$20K=24$$

$$K=\frac{6}{5}$$

$$y_{Ö}(n)=K4^n u(n)=\frac{6}{5}4^n u(n)$$

$$y_z(n)=y_d(n)+y_{Ö}(n)=c_3(-1)^n+c_4 4^n+\frac{6}{5}n4^n u(n)$$

$$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$$

$$\left.\begin{array}{l} y(0)-3y(-1)-4y(-2)=x(0)+2x(-1)\Rightarrow y(0)=1=c_3+c_4+0 \\ y(1)-3y(0)-4y(-1)=x(1)+2x(0)\Rightarrow y(1)=9=-c_3+4c_4+\frac{24}{5} \end{array}\right\} \begin{array}{l} c_3=-\frac{1}{25} \\ c_4=\frac{26}{25} \end{array}$$

$$y_T(n)=y_d(n)+y_{Ö}(n)=[-\frac{1}{25}(-1)^n+\frac{26}{25}(4)^n+\frac{6}{5}n4^n]u(n)$$

Örnek

$y(n)+0,5y(n-1)=x(n) \qquad n\geq 0$

$y(-1)=2 \qquad x(n)=u(n)$ toplam çözümü bulunuz?

Çözüm

$y(n)=\lambda^n$

$$y(n)+0,5y(n-1)=x(n)$$
$$\lambda^n+0,5\lambda^{n-1}=0$$
$$\lambda^{n-1}\underbrace{(\lambda+0,5)}_{\lambda_1=0,5}=0$$

$$y_d(n)=c_1\lambda^n$$
$$y(0)-0,5y(-1)=0$$
$$y(0)=-1$$
$$-1=c_1$$

$$y_d(n)=c_1\lambda^n=-1(-\frac{1}{2})^n$$

$$y_z(n)=y_d(n)+y_Ö(n)$$
$$=c_2\lambda^n+Ku(n)$$
$$=(-\frac{1}{2})^n+Ku(n)$$

$$y(n)+0,5y(n-1)=x(n)$$
$$Ku(n)+\frac{1}{2}Ku(n-1)=u(n)$$
$$K+\frac{1}{2}K=1$$
$$K=\frac{2}{3}$$

$$y(0)+0,5\underbrace{\cancel{y(-1)}}_{2}=x(0)$$
$$y(0)=1=c_2+\frac{2}{3}$$

$$y_z(n)=y_d(n)+y_ö(n)=\frac{1}{3}(-\frac{1}{2})^n+\frac{2}{3}u(n)$$

$$y_T(n)=y_d(n)+y_z(n)$$
$$=-(-\frac{1}{2})^n+\frac{1}{3}(-\frac{1}{2})^n+\frac{2}{3}u(n)$$
$$=-\frac{2}{3}(-\frac{1}{2})^n+\frac{2}{3}u(n)$$

Birim impuls cevabının hesaplanması

$x(n)=\delta(n)$ uygulanarak elde edilen zorlanmış çözümdür.

$n>0$ olduğunda $x(n)=0$ olacağından $y_{Ö}(n)=0$ olur

Birim impuls cevabı sadece $y_d(n)$ ve **0** başlangıç koşulları kullanılarak bulunur.

Örnek $y(n)+0,5y(n-1)=x(n)$ fark denkleminin birim impuls cevabını hesaplayınız?

Çözüm

$$x(n)=\delta(n) \qquad y(n)=h(n)$$

$$y(n)+0,5y(n-1)=x(n)$$
$$h(n)+0,5h(n-1)=\delta(n)$$
$$h(0)+0,5\underbrace{h(-1)}_{0 \text{ kabul edilir}}=\delta(0)$$
$$h(0)=1=c_1$$

$$y_d(n)=c1(-\frac{1}{2})^n=h(n)$$
$$h(n)=(-\frac{1}{2})^n u(n)$$

Örnek $y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$ fark denkleminin birim impuls cevabını hesaplayınız $h(n)=?$

Çözüm

$$x(n)=\delta(n) \qquad y(n)=h(n)$$

$$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$$
$$h(n)-3h(n-1)-4h(n-2)=\delta(n)+2\delta(n-1)$$
$$h(0)-3h(-1)-4h(-2)=\delta(0)+2\delta(-1)$$
$$h(0)=1=c_1(-1)^0+c_2(4)^0$$
$$h(1)-3h(0)-4h(-1)=\delta(1)+2\delta(0)$$
$$h(1)=5=c_1(-1)^1+c_2 1(4)^1$$

$$1=c_1+c_2$$
$$5=-c_1+4c_2$$
$$c_1=\frac{6}{5} \quad c_2=\frac{-1}{5}$$

$$h(n)=y_d(n)$$
$$=(c_1(-1)^n+c_2(4)^n)u(n)$$
$$=(\frac{-1}{5}(-1)^n+\frac{6}{5}(4)^n)u(n)$$

Durum Değişkenleri

Sistemin içerisindeki değişkenler

Bizim müdahale edemediğimiz değişkenler

$$y(n) = \sum_{k=0}^{N} a_k x(n-k) - \sum_{k=0}^{N} b_k y(n-k)$$

$$= x(n) + \frac{1}{2} x(n-1)$$

İşlem adımları;

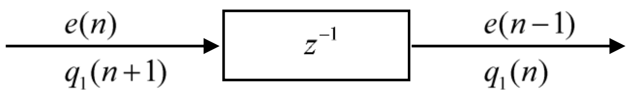
1. Fark denkleminin $y(n)$ gördüğümüz yere $e(n)$ yazıyoruz
2. Eşitliğin sağ tarafında ne yazdığından bağımsız olarak (ne olursa olsun) $x(n)$ yazıyoruz

   $$e(n) + \frac{1}{2} e(n-1) = x(n)$$

   $$e(n) = x(n) - \frac{1}{2} e(n-1)$$

3. Sistemin derecesi ne ise durum değişkenlerinin derecesi de o olur. Bu örnekte 1 değişken var (1.dereceden)

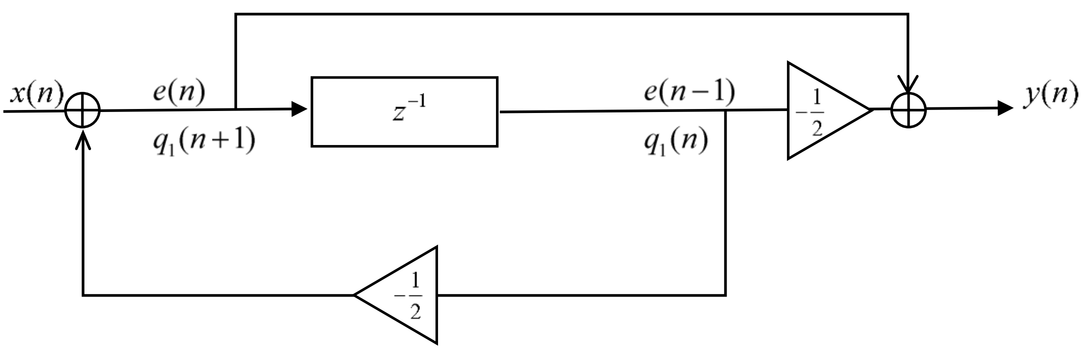
   $q_1(n) = e(n-1)$ // ***Durum değişkenleri***



$q_1(n+1)$ // $q_1(n)$ kullanarak yazıyoruz

$$q_1(n+1) = e(n)$$

$$= x(n) - \frac{1}{2} q_1(n)$$



$$y(n) = e(n)$$

$$= x(n) - \frac{1}{2} q_1(n)$$

Durum değişkenleri yöntemi

Fark denklemiyle modellenen nedensel süzgeçlerin iç değişkenlerinin durumunu belirlemek için durum değişkenleri yaklaşımı kullanılır. Sistemin tüm durum değişkenleri durum vektörü adı verilen bir vektörle gösterilir. Durum değişkenleri N nci dereceden fark denklemini N adet birinci dereceden sisteme dönüştürerek elde edilir. Bu amaçla, aşağıdaki N nci dereceden fark denklemini ele alalım.

$$y(n)=\sum_{k=0}^{N}a_k.x(n-k)-\sum_{k=1}^{N}b_k.y(n-k)$$

Bu süzgeci birbirine seri bağlanmış iki süzgece ayırabiliriz.

$$\omega(n)=x(n)-\sum_{k=1}^{N}b_k.\omega(n-k) \qquad y(n)=\sum_{k=0}^{N}a_k.\omega(n-k)$$

Bu ifadeleri yeniden düzenlenmek suretiyle fark denklemi elde edilir.

$q_1(n)$, $q_2(n)$, ….. , $q_N(n)$ durum değişkenleri de aşağıdaki gibi tanımlanır.

$$\begin{aligned}
q_1(n)&=\omega(n-N)\\
q_2(n)&=\omega(n-N+1)\\
&\vdots\\
q_{N-1}(n)&=\omega(n-2)\\
q_N(n)&=\omega(n-1)
\end{aligned}$$

denklemlerinden durum değişkenleri arasındaki ilişki yazılabilir.

$$\begin{aligned}
q_1(n+1)&=\omega(n-N+1)=q_2(n)\\
q_2(n+1)&=\omega(n-N+2)=q_3(n)\\
&\vdots\\
q_{N-1}(n+1)&=\omega(n-1)=q_N(n)\\
q_N(n+1)&=\omega(n)=x(n)-b_1\omega(n-1)-b_2\omega(n-2)-\ldots\ldots-b_N\omega(n-N)\\
&=x(n)-b_1q_N(n)-b_2q_{N-1}(n)-\ldots\ldots-b_Nq_1(n)
\end{aligned}$$

Bu matrisleri denklem formunda gösterebiliriz.

$$\underbrace{\begin{bmatrix} q_1(n+1)\\ q_2(n+1)\\ \vdots\\ q_{N-1}(n+1)\\ q_N(n+1)\end{bmatrix}}_{q(n+1)}=\underbrace{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0\\ 0 & 0 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 0 & 1\\ -b_N & -b_{N-1} & -b_{N-2} & \cdots & -b_2 & -b_1\end{bmatrix}}_{A}\cdot\underbrace{\begin{bmatrix} q_1(n)\\ q_2(n)\\ \vdots\\ q_{N-1}(n)\\ q_N(n)\end{bmatrix}}_{q(n)}+\underbrace{\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}}_{B}\cdot x(n)$$

$\omega(n)$ değişkeni yok edilebilir.

$$y(n) = a_0 . \left[x(n) - \sum_{k=1}^{N} b_k . \omega(n-k) \right] + \sum_{k=1}^{N} a_k . \omega(n-k)$$
$$= a_0 x(n) + \sum_{k=1}^{N} \left[a_k - a_0 b_k \right] . \omega(n-k)$$

Aşağıdaki katsayıları tanımlayalım.

$$c_1 = a_N - a_0 b_N$$
$$c_2 = a_{N-1} - a_0 b_{N-1}$$
$$\vdots$$
$$c_{N-1} = a_2 - a_0 b_2$$
$$c_N = a_1 - a_0 b_1$$

çıkış ifadesi,

$$y(n) = a_0 x(n) + c_1 q_1(n) + c_2 q_2(n) + c_3 q_3(n) + \ldots\ldots + c_{N-1} q_{N-1}(n) + c_N q_N(n)$$

veya

$$y(n) = \underbrace{\begin{bmatrix} c_1 & c_2 & \cdots & c_N \end{bmatrix}}_{C} \cdot \begin{bmatrix} q_1(n) \\ q_2(n) \\ \vdots \\ q_N(n) \end{bmatrix} + \underbrace{\begin{bmatrix} a_0 \end{bmatrix}}_{d} x(n)$$

olarak yazılabilir. Girişine $x(n)$ işareti uygulanan doğrusal bir sistemin çıkışı $y(n)$ olduğuna göre, durum denklemleri aşağıdaki gibi yazılabilir.

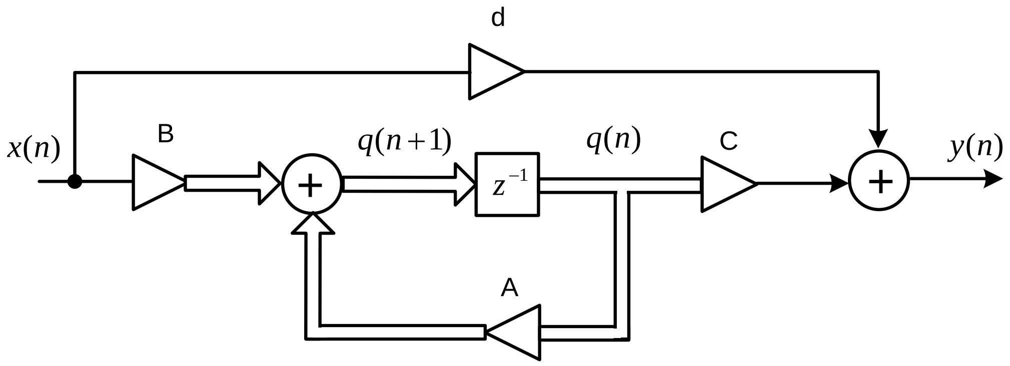
$$q(n+1) = Aq(n) + Bx(n)$$

$$y(n) = Cq(n) + dx(n)$$

A sistem matrisi, B kontrol vektörü, C gözlem vektörü ve d geçiş katsayısı olarak kullanılır. A matrisi N nci dereceden bir kare matristir. B ve C vektörleri N boyutludur. $q(n)$ ise durum değişkenleri içeren durum vektörüdür.

$$q(n) = \begin{bmatrix} q_1(n) & q_2(n) & \cdots & q_N(n) \end{bmatrix}^T$$

Şekilde durum değişkenlerine ilişkin blok diyagramı gösterilimi verilmiştir. Burada çift çizgiler vektör işaretleri göstermektedir.



Şekil Durum değişkenleri yöntemiyle modellenen süzgecin blok diyagramı

Örnek Sayısal bir süzgeç aşağıdaki fark denklemiyle tanımlansın:

$$y(n) = x(n) + 2x(n-1) + x(n-2) - y(n-1) + 2y(n-2)$$

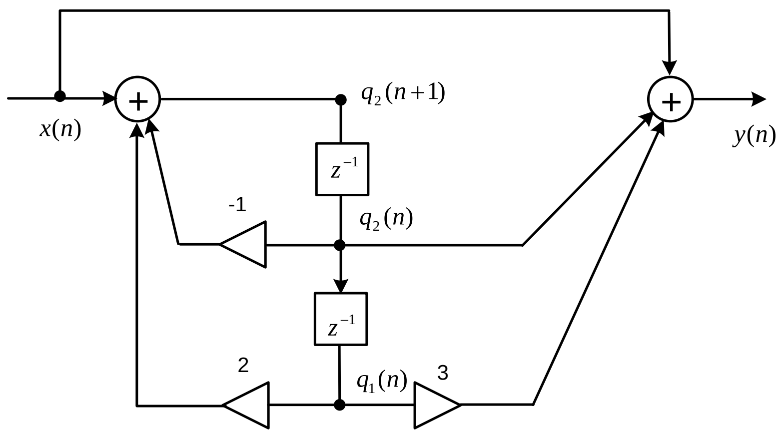
Yukarıdaki denkleminden $a_0 = 1$, $a_1 = 2$, $a_2 = 1$, $b_0 = 1$, $b_1 = 1$, $b_2 = -2$ olarak belirlendiğinden, bu süzgeç durum değişkenleri yöntemi ile aşağıdaki gibi gösterilir.

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot x(n)$$

$$c_1 = a_2 - a_0 b_2 = 1 - 1(-2) = 3$$
$$c_2 = a_1 - a_0 b_1 = 2 - 1.1 = 1$$

bulunur. O halde çıkış, durum değişkenleri ve giriş cinsinden aşağıdaki gibi verilir.

$$y(n) = \begin{bmatrix} 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + x(n)$$



Şekil Örnek 2.11 deki sayısal süzgecin durum denklemleri cinsinden blok diyagramı

Örnek

$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$ durum değişkenleri?

A,B,C,D bulunduğunda çözüm bulunmuş olur

Çözüm

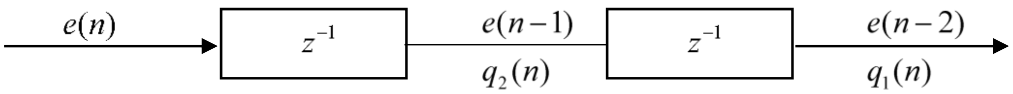
$$\overset{b0}{1}\,y(n)-\overset{b1}{3}\,y(n-1)-\overset{b2}{4}\,y(n-2)=\overset{a0}{1}\,x(n)+\overset{a1}{2}\,x(n-1)$$

$$e(n)-3e(n-1)-4e(n-2)=x(n)$$

2. Dereceden sistem

Durum değişkenleri;

$$q_1(n)=e(n-2) \qquad \alpha(n)=\begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix}$$
$$q_2(n)=e(n-1)$$



Durum denklemleri;

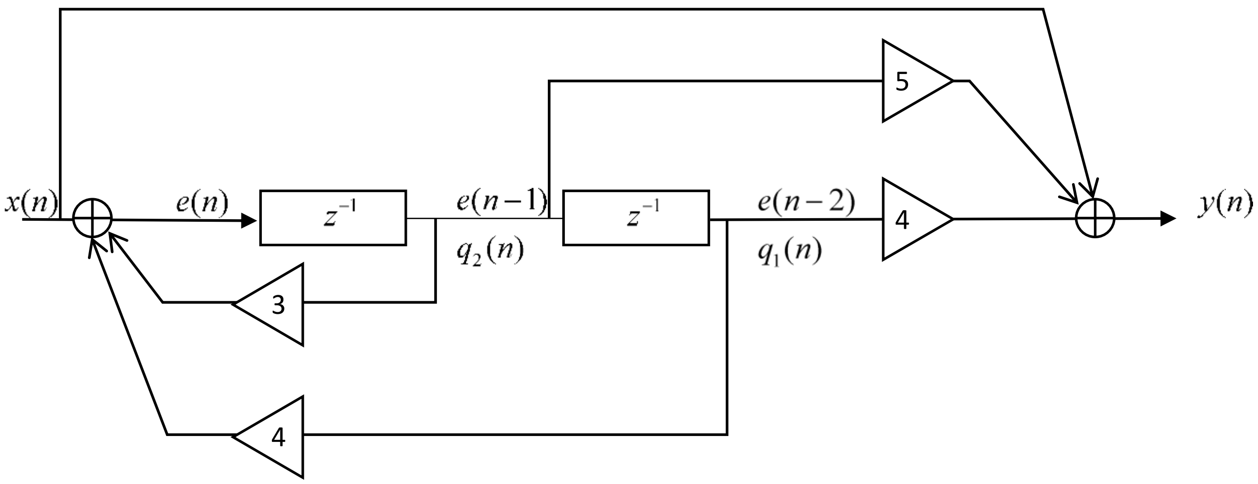
$$q_1(n+1)=e(n-1)=q_2(n)$$
$$q_2(n+1)=e(n)=x(n)+3q_2(n)+4q_1(n)$$

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \end{bmatrix}=\underbrace{\begin{bmatrix} \overset{q_1}{0} & \overset{q_2}{1} \\ 4 & 3 \end{bmatrix}}_{A}\begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix}+\underbrace{\begin{bmatrix} \overset{x_n}{0} \\ 1 \end{bmatrix}}_{B}x(n)$$

1.denklemdeki q katsayıları

X(n)'nin katsayısı

2.denklemdeki q katsayıları



$$y(n)=e(n)+2e(n-1)$$
$$=\underbrace{x(n)+3e(n-1)+4e(n-2)}_{e(n)}+2e(n-1)$$
$$=x(n)+5e(n-1)+4e(n-2)$$
$$=x(n)+5q_2(n)+4q_1(n)$$

$$y(n)=\underbrace{\begin{bmatrix} \overset{q_1}{4} & \overset{q_2}{5} \end{bmatrix}}_{\substack{C \\ \text{katsayılar } y(n)\text{'de}}}\begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix}+\underset{\substack{D \\ \text{katsayılar } y(n)\text{'de}}}{\begin{bmatrix} \overset{x_n}{1} \end{bmatrix}}\;x(n)$$

Z Dönüşümü

z bir karmaşık sayı

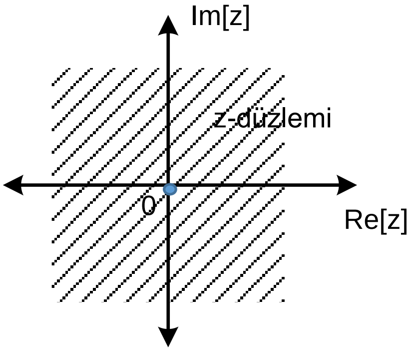
$$\sigma + j\omega = re^{j\theta}$$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

Örnek **Sınırlı eleman varsa/Sağ taraflı** dizi $x(n) = \left\{ \underset{\underset{0}{\uparrow}}{1}, \underset{\underset{1}{\perp}}{2}, \underset{\underset{2}{\perp}}{5}, \underset{\underset{3}{\perp}}{7}, \underset{\underset{4}{\perp}}{0}, \underset{\underset{5}{\perp}}{1} \right\}$ $x(z) = ?$

Çözüm

$$x(z) = x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4} + x(5)z^{-5}$$
$$= 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + 0z^{-4} + 1z^{-5}$$



Yakınsama Bölgesi

$z \neq 0$ 'da karmaşık sayıları içerisinde barındıran bölge

$z \neq 0$ 'da ∞'a gider, bu hariç her yer YB (Yakınsama bölgesi)

Örnek **Sınırlı eleman varsa/Sol taraflı** dizi $x(n) = \left\{ \underset{\underset{-5}{\perp}}{1}, \underset{\underset{-4}{\perp}}{2}, \underset{\underset{-3}{\perp}}{5}, \underset{\underset{-2}{\perp}}{7}, \underset{\underset{-1}{\perp}}{0}, \underset{\underset{0}{\uparrow}}{1} \right\}$ $x(z) = ?$

Çözüm

$$x(z) = x(0)z^{0} + x(-1)z^{1} + x(-2)z^{2} + x(-3)z^{3} + x(-4)z^{4} + x(-5)z^{5}$$
$$= 1z^{0} + 0z^{1} + 7z^{2} + 5z^{3} + 2z^{4} + 1z^{5}$$
$$= 1 + 7z^{2} + 5z^{3} + 2z^{4} + z^{5}$$

Yakınsama Bölgesi (YB)

$z \neq \infty$ olduğu yerlerde YB (Yakınsama bölgesi)

Örnek **Sınırlı eleman varsa/Sağ ve Sol taraflı** $x(n) = \left\{ \underset{\underset{-2}{\perp}}{1}, \underset{\underset{-1}{\perp}}{2}, \underset{\underset{0}{\uparrow}}{5}, \underset{\underset{1}{\perp}}{7}, \underset{\underset{2}{\perp}}{0}, \underset{\underset{3}{\perp}}{1} \right\}$ $x(z) = ?$

Çözüm

$$x(z) = x(-2)z^{2} + x(-1)z^{1} + x(0)z^{0} + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}$$
$$= 1z^{2} + 2z^{1} + 5z^{0} + 7z^{-1} + 0z^{-2} + 1z^{-3}$$
$$= z^{2} + 2z + 5 + 7z^{-1} + z^{-3}$$

Yakınsama Bölgesi (YB)

$z \neq \infty \quad \cap \quad z \neq 0$ olduğu yerlerde YB (Yakınsama bölgesi)

Örnek

$x(n) = \delta(n) \qquad x(z) = ? \qquad YB = ?$

Çözüm

$x(n) = \left\{ \underset{0}{\underset{\uparrow}{1}} \right\} \qquad x(z) = 1 \qquad YB = \text{Tüm karmaşık düzlem}$

Örnek

$x(n) = \delta(n-2) \qquad x(z) = ? \qquad YB = ?$

Çözüm

$x(n) = \left\{ \underset{2}{\underset{\uparrow}{1}} \right\} \qquad x(z) = 1z^{-2} \qquad YB \quad n \neq 0 \quad 0'dan \text{ farklı tüm karmaşık düzlem}$

Örnek

$x(n) = \delta(n+2) \qquad x(z) = ? \qquad YB = ?$

Çözüm

$x(n) = \left\{ \underset{-2}{\underset{\uparrow}{1}} \right\} \qquad x(z) = 1z^{2} \qquad YB \quad n \neq \infty$

Örnek

$x(n) = \alpha^n u(n) \qquad x(z) = ? \qquad YB = ?$

Çözüm

$$\begin{aligned} x(n) &= \alpha^n u(n) \\ &= \sum_{n=0}^{\infty} \alpha^n z^{-n} \\ &= \sum_{n=0}^{\infty} (\alpha z^{-1})^n \\ &= \frac{1}{1-\alpha z^{-1}} \end{aligned}$$

$$\begin{aligned} x(n) &= -\alpha^n u(-n-1) \\ &= \frac{1}{1-\alpha z^{-1}} \end{aligned}$$

$YB \quad |z| < |\infty|$

$YB \quad |\alpha z^{-1}| < 1 \qquad |z| > |\infty|$

Örnek

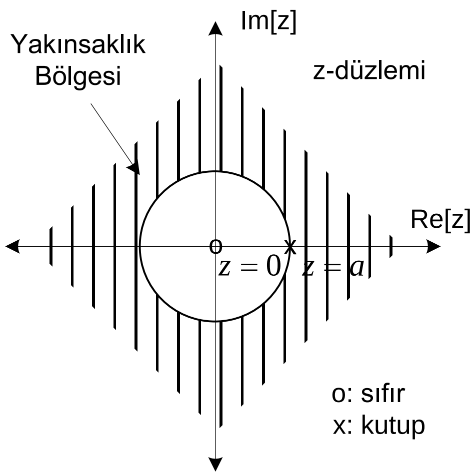
Sağ taraflı üstel $x(n) = a^n u(n)$ dizisi için z -dönüşümü aşağıdaki gibi yazılır.

$$X(z) = \sum_{n=0}^{\infty} a^n . z^{-n} = \sum_{n=0}^{\infty} (a.z^{-1})^n$$

Burada $\left|a.z^{-1}\right| < 1$ için seri yakınsak olur ve z -dönüşümü aşağıdaki gibi bulunur.

$$X(z) = \frac{1}{1 - a.z^{-1}} = \frac{z}{z - a}$$

$\left|a.z^{-1}\right| < 1$ koşulundan $|z| > |a|$ yazılabilir. Yakınsaklık bölgesi a yarıçaplı dairenin dışında kalan bölgedir. $X(z)$ nin $z = 0$ da bir sıfırı ve $z = a$ da bir kutbu vardır.



Şekil $x(n) = a^n u(n)$ dizisi için sıfır-kutup diyagramı ve yakınsaklık bölgesi

Örnek

Sol taraflı bir diziye örnek olarak aşağıdaki diziyi ele alalım.

$$x(n)=\begin{cases}0, & \text{n} \geq 0 \quad \text{için} \\ -b^n, & \text{n} \leq -1 \quad \text{için}\end{cases}$$

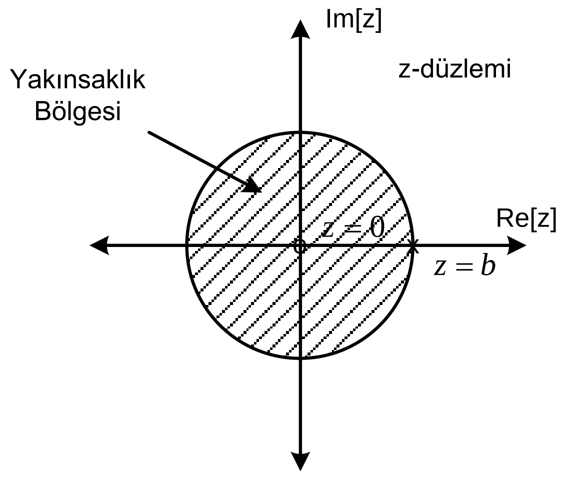
$x(n)$ nin z -dönüşümü için aşağıdaki ifade yazılabilir.

$$X(z)=\sum_{n=-\infty}^{-1}-b^n.z^{-n}=\sum_{n=1}^{\infty}-b^{-n}.z^n=1-\sum_{n=0}^{\infty}b^{-n}.z^n=1-\sum_{n=0}^{\infty}(b^{-1}.z)^n$$

Eğer $\left|b^{-1}.z\right|<1$ veya $|z|<b$ ise (6.12) deki seri yakınsar.

$$X(z)=1-\frac{1}{1-b^{-1}.z}=\frac{-b^{-1}.z}{1-b^{-1}.z}=\frac{z}{-b+z}=\frac{z}{z-b}$$

Yakınsaklık bölgesi b yarıçaplı dairenin içinde kalan alandır.



Şekil $x(n)=-b^n u(-n-1)$ dizisi için sıfır-kutup diyagramı ve yakınsaklık bölgesi

<u>Açıklama</u> Son iki örnekteki dizilere ait z -dönüşümlerinin incelenmesinden, sadece z -dönüşümünün sıfırları ve kutupları yardımıyla dizileri belirlemenin mümkün olmadığı görülmektedir. Gerçekten $a=b$ olması halinde, sağ ve sol taraflı dizilerin z -dönüşümleri aynı olmaktadır. Farklı olan özellik ise yakınsaklık bölgeleridir. O halde, diziyi belirlerken z -dönüşümünün yanı sıra yakınsaklık bölgesi de verilmelidir. Dizinin sağ veya sol taraflı olarak belirtilmesi durumunda da yakınsaklık bölgesi dolaylı olarak verilmiş olur.

Örnek

İki taraflı diziye örnek olarak

$$x(n) = \begin{cases} a^n, & \text{n} \geq 0 \quad \text{için} \\ -b^n, & \text{n} < 0 \quad \text{için} \end{cases}$$

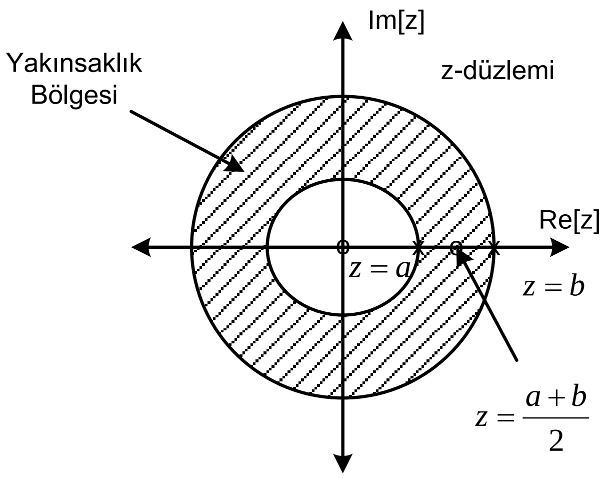
dizisinin z -dönüşümünü bulalım.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n).z^{-n} = \sum_{n=-\infty}^{-1} -b^n.z^{-n} + \sum_{n=0}^{\infty} a^n.z^{-n}$$

$|a.z^{-1}| < 1$ ve $|b^{-1}.z| < 1$ koşullarının sağlanması durumunda,

$$X(z) = \frac{z}{z-b} + \frac{z}{z-a} = \frac{z(2z-a-b)}{(z-a).(z-b)}$$

şeklinde yazılabilir. Yakınsaklık bölgesi şekildeki gibi yarıçapları a ve b olan halka içindedir. Yani, $|a| < |b|$ ise, $|a| < |z| < |b|$ yakınsaklık bölgesidir.



Şekil $x(n) = a^n u(n) - b^n u(-n-1)$ dizisi için sıfır-kutup diyagramı ve yakınsaklık bölgesi

Standart z -Dönüşümleri

Dizi	z -Dönüşümü	Yakınsaklık Aralığı
$\delta(n)$	1	Tüm z
$\delta(n-m)$, $m>0$	z^{-m}	$\|z\|>0$, yani $z=0$ hariç tüm z
$\delta(n+m)$, $m>0$	z^{m}	$\|z\|<\infty$, yani $z=\infty$ hariç tüm z
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\|>1$
$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\|z\|<1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
$u(n)\cos n\theta$	$\frac{1-z^{-1}\cos\theta}{1-2z^{-1}\cos\theta+z^{-2}}$	$\|z\|>1$
$u(n)\sin n\theta$	$\frac{z^{-1}\sin\theta}{1-2z^{-1}\cos\theta+z^{-2}}$	$\|z\|>1$
$u(n)r^n\cos n\theta$	$\frac{1-rz^{-1}\cos\theta}{1-2rz^{-1}\cos\theta+r^2z^{-2}}$	$\|z\|>\|r\|$
$u(n)r^n\sin n\theta$	$\frac{rz^{-1}\sin\theta}{1-2rz^{-1}\cos\theta+r^2z^{-2}}$	$\|z\|>\|r\|$

Örnek $x(n)=\left(\frac{1}{2}\right)^n\left(u(n)-u(n-10)\right)$ $X(z)=?$

Çözüm

$$X(z)=\frac{1}{1-\alpha z^{-1}}$$

$$=\frac{1}{1-\left(\frac{1}{2}\right)z^{-1}} \qquad YB \quad z\neq 0$$

$$x(n)=\left\{\underset{0}{\underset{\uparrow}{1}},\underset{1}{\frac{1}{2}},\underset{2}{\frac{1}{4}},\underset{3}{\frac{1}{8}},\cdots\underset{9}{\left(\frac{1}{2}\right)^9}\right\}$$ sağ taraflı ve sınırlı

$$X(z)=1z^0+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}+\cdots+\left(\frac{1}{2}\right)^9 z^{-9}$$

Ödev $x(n)=\left(\frac{1}{2}\right)^n u(n)-\left(\frac{1}{2}\right)^n u(n-10)$ $X(z)=?$

Örnek $x(n)=(2)^n u(n)$ $X(z)=?$ $YB=?$

Çözüm

$$X(z)=\frac{1}{1-\alpha z^{-1}}=\frac{1}{1-2z^{-1}} \qquad YB \quad |z|>2$$

Örnek $x(n)=(-2)^n u(n)$ $X(z)=?$ $YB=?$

Çözüm

$$X(z)=\frac{1}{1-\alpha z^{-1}}=\frac{1}{1-(-2)z^{-1}}=\frac{1}{1+2z^{-1}} \qquad YB \quad |z|>2$$

Örnek $x(n)=\delta(n)$ $X(z)=?$ $YB=?$

Çözüm

$X(z)=1$ YB Tüm z (karmaşık) düzlem

Z dönüşümü özellikleri

1. Doğrusallık (z dönüşümü)

$$x_1(n) \quad ve \quad x_2(n)$$

$$x_1(n) \leftrightarrow X_1(z) \qquad YB1$$

$$x_2(n) \leftrightarrow X_2(z) \qquad YB2$$

$$x_3(n) = ax_1(n) + bx_2(n)$$

$$X_3(z) = aX_1(z) + bX_2(z) \qquad YB1 \cap YB2$$

Örnek $x(n) = \left[3(2)^n - 4(3)^n\right]u(n)$ şeklinde verilen dizinin $X(z) = ?$ $YB = ?$

Çözüm

$$X(z) = 3 \cdot \frac{1}{1-2z^{-1}} - 4 \cdot \frac{1}{1-3z^{-1}} = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}}$$

$$YB \quad |z| > 2 \ \cap \ |z| > 3 \qquad |z| > 3$$

Örnek $x(n) = \cos(\omega_0 n)u(n)$ şeklinde verilen dizinin $X(z) = ?$ $YB = ?$

Çözüm

$$X(z) = \frac{1 - z^{-1}\cos(\omega_0 n)}{1 - 2z^{-1}\cos(\omega_0 n) + z^{-2}} \qquad YB \quad |z| > 1$$

Sağ taraflı ve sınırsız (Çemberin dışı)

$$\cos\omega_0 n = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2} = \frac{1/2}{1 - e^{j\omega_0 n}z^{-1}} + \frac{1/2}{1 - e^{-j\omega_0 n}z^{-1}}$$

$$\left|e^{j\omega_0}\right| \qquad z = r.e^{j\theta} \qquad |z| = r$$

$$|z| > 1 \ \cap \ |z| > 1 \qquad |z| > 1$$

2. Öteleme (z dönüşümü)

a.

$$x(n) \leftrightarrow X(z)$$
$$x_1(n) \leftrightarrow x(n-k)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\cdot z^{-n}$$
$$= \cdots + x(-1)z^{1} + x(0)z^{0} + x(1)z^{-1} + \cdots$$
$$= \cdots + x(-1)z + x(0) + x(1)z^{-1} + \cdots$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1(n)\cdot z^{-1}$$
$$= \sum_{n=-\infty}^{\infty} x_1(n-k)\cdot z^{-n}$$
$$= \cdots + x(-1)z^{-k+1} + x(0)z^{-k} + x(1)z^{-(k+1)} + \cdots$$
$$= z^{-k}X(z)$$

Öteleme sağ tarafa ise $X_1(z) = z^{-k}X(z)$

$$x(n) \leftrightarrow X(z)$$
$$x_2(n) \leftrightarrow x(n+k)$$
$$X_2(z) = \sum_{n=-\infty}^{\infty} x_2(n)\cdot z^{-1}$$
$$= \sum_{n=-\infty}^{\infty} x_2(n+k)\cdot z^{-n}$$
$$= z^{k}X(z)$$

Öteleme sol tarafa ise $X_2(z) = z^{k}X(z)$

Örnek $x(n) = 2^n u(n-2)$ şeklinde verilen dizinin $X(z) = ?$ $YB = ?$

Çözüm

$$x(n) = 2^n u(n-2)$$
$$= 2^2\cdot 2^{-2}\cdot 2^n u(n-2)$$
$$= 4\cdot 2^{n-2}u(n-2)$$
$$X(z) = 4\cdot\frac{z^{-2}}{1-2z^{-1}}$$

YB

$|z| > 2$

b. $x(-n) = X\left(z^{-1}\right)$

Örnek $x(n) = 2^{-n}u(-n-2)$ şeklinde verilen dizinin $X(z) = ?$ $YB = ?$

Çözüm

$$\begin{aligned} x(n) &= 2^{-n}u(-n-2) \\ &= 2^{2}\cdot 2^{-2}\cdot 2^{-n}u(-n-2) \\ &= 4\cdot 2^{-n-2}u(n-2) \end{aligned}$$

$$X(z) = 4\cdot\frac{z^2}{1-2z}$$

YB

$|z| < 2$

c. $x(n) = X\left(z\right)$

$x_1(n) = n\,x(n)$

$$\begin{aligned} X_1(z) &= -z\frac{d}{dz}X(z) \\ &= -z\left(-\sum_{n=-\infty}^{\infty} n\cdot x(n)\cdot z^{-n-1}\right) \end{aligned}$$

$$\begin{aligned} \frac{d}{dn}\left(\sum_{n=-\infty}^{\infty} x(n)\cdot z^{-n}\right) &= \sum_{n=-\infty}^{\infty} x(n)\cdot\left(-n\cdot z^{-n-1}\right) \\ &= \sum_{n} n\cdot x(n)\cdot z^{-n}z^{-1} \end{aligned}$$

$$x(n) \leftrightarrow X(z)$$

$$n\cdot x(n) \leftrightarrow -z\frac{d}{dz}X(z)$$

3. Konvolüsyon (z dönüşümü)

$$y(n) = x(n) * h(n)$$
$$Y(z) = X(z) \cdot H(z)$$

Transfer Fonksiyonu; $H(z) = \dfrac{Y(z)}{X(z)}$

Örnek $y(n) = \dfrac{1}{2} y(n-1) + 2x(n)$ şeklinde verilen dizinin $H(z) = ?$ $YB = ?$

Çözüm

$$y(n) = \frac{1}{2} y(n-1) + 2x(n)$$
$$Y(z) - \frac{1}{2} z^{-1} Y(z) = 2X(z)$$
$$Y(z)(1 - \frac{1}{2} z^{-1}) = 2X(z)$$
$$\frac{Y(z)}{X(z)} = \frac{2}{1 - \frac{1}{2} z^{-1}}$$

YB

$$|z| > \frac{1}{2}$$

Örnek

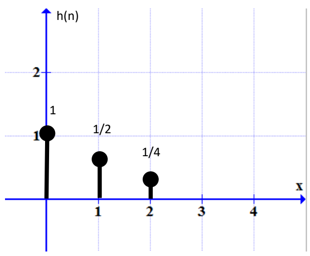
$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \le n \le 2 \\ 0 & diğer \end{cases}$$

$x(n) = \delta(n) + \delta(n-1) + 4\delta(n-2)$ $y(n) = ?$ $YB = ?$

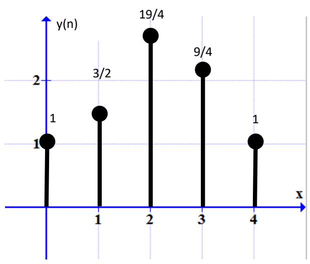
Çözüm

$$x(n) = \delta(n) + \delta(n-1) + 4\delta(n-2)$$
$$X(z) = 1 \cdot z^0 + 1 \cdot z^{-1} + 4 \cdot z^{-2}$$

$$h(n) = \left\{1, \frac{1}{2}, \frac{1}{4}\right\}$$
$$H(z) = 1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}$$



$$Y(z) = X(z) \cdot H(z)$$
$$= \left(1 + z^{-1} + 4z^{-2}\right)\left(1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}\right)$$
$$= 1 + \frac{3}{2} z^{-1} + \frac{19}{4} z^{-2} + \frac{9}{4} z^{-3} + z^{-4}$$
$$y(n) = \delta(n) + \frac{3}{2}\delta(n-1) + \frac{19}{4}\delta(n-2) + \frac{9}{4}\delta(n-3) + \delta(n-4)$$



Örnek $x(n) = n\left(\frac{1}{2}\right)^n u(n-2)$ $X(z) = ?$ $YB = ?$

Çözüm

$$x_1(n) = \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{-2} \cdot \left(\frac{1}{2}\right)^n u(n-2)$$

$$= \left(\frac{1}{2}\right)^2 \cdot \underbrace{\left(\frac{1}{2}\right)^{n-2} u(n-2)}_{x_2(n)}$$

$$x_2(n) = \left(\frac{1}{2}\right)^{n-2} u(n-2)$$

$$x_3(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$X_3(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$x_2(n) = x_3(n-2)$$

$$X_2(z) = z^{-2} X_3(z)$$

$$= z^{-2} \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$= \frac{z^{-2}}{1-\frac{1}{2}z^{-1}}$$

$$X_1(z) = \left(\frac{1}{2}\right)^2 \cdot X_2(z)$$

$$= \frac{1}{4} \cdot \frac{z^{-2}}{1-\frac{1}{2}z^{-1}}$$

$$x(n) = n\left(\frac{1}{2}\right)^n u(n-2)$$

$$X(z) = -z\frac{d}{dx}(X_1(z))$$

$$= -z\frac{d}{dx}\left(\frac{1}{4} \cdot \frac{z^{-2}}{1-\frac{1}{2}z^{-1}}\right)$$

$$= -z\frac{-\frac{1}{2}z^{-3}\left(1-\frac{1}{2}z^{-1}\right) - \frac{1}{2}z^{-2} \cdot \frac{1}{4}z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)^2}$$

YB

$$|z| > \frac{1}{2}$$

Örnek $x(n) = |n|\left(\frac{1}{2}\right)^{|n|}$ $X(z) = ?$ $YB = ?$

Çözüm

$$x(n) = \underbrace{n\left(\frac{1}{2}\right)^n u(n)}_{x_1(n)} - \underbrace{n\left(\frac{1}{2}\right)^{-n} u(-n-1)}_{x_2(n)}$$

Çift taraflı

$(0, \infty)$ – $(-1, -\infty)$

$$X(z) = -z\frac{d}{dz}(X_1(z)) - \left(-z\frac{d}{dz}(X_2(z))\right)$$

$$= -z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) + z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

YB

$$|z| > \frac{1}{2} \quad \cap \quad |z| < 2$$

$$\frac{1}{2} < |z| < 2$$

Ters Z dönüşümü

1.Yöntem

Counchy Entegral (Rezidü teoremi)

$$X(z)=\frac{1}{1-\alpha z^{-1}} \qquad |z|>|\alpha|$$

$$x(n)=\sum_{\text{tüm kutuplar}} \operatorname{Res}\left(X(z)z^{n-1}\right)$$

$$\operatorname{Res}\left(z-z_i\right)x(z)z^{n-1}\Big|_{z=z_i}$$

Örnek $X(z)=\frac{1}{1-\alpha z^{-1}}$ *YB* $|z|>|\alpha|$ Ters z dönüşümünü bulunuz?

Çözüm

$$x(z)z^{n-1}=\frac{z^{n-1}}{1-\alpha z^{-1}}=\frac{z^n}{z-\alpha}\Bigg|_{z=\alpha}$$

$$x(n)=\operatorname{Res}\left(z-z_i\right)x(z)z^{n-1}\Big|_{z=z_i}=\frac{z^n}{\cancel{z-\alpha}}\cdot\cancel{(z-\alpha)}\Bigg|_{z=\alpha}=\alpha^n \qquad n\geq 0 \textit{ olduğu sürece}$$

$$x(n)=\alpha^n \qquad n=-1\cdot X(z)\cdot z^{n-1}=\frac{1}{z(z-\alpha)} \qquad n<0 \textit{ olursa}$$

$$\frac{1}{\underset{z=0}{z}\underbrace{(z-\alpha)}_{z=\alpha}}\left\}\begin{matrix}\operatorname{Res} zX(z)z^{n-1}\Big|_{z=0}=\cancel{z}\frac{1}{\cancel{z}(z-\alpha)}\Bigg|_{z=0}=-\frac{1}{\alpha}\\ \operatorname{Res}(z-\alpha)X(z)z^{n-1}\Big|_{z=\alpha}=\frac{1}{z}\Big|_{z=\alpha}=\frac{1}{\alpha}\end{matrix}\right\rangle \quad x(n)=-\frac{1}{\alpha}+\frac{1}{\alpha}=0 \qquad n<0$$

$$\alpha^n \rightarrow x(n)=\alpha^n u(n)$$

Ters Z dönüşümü

2.Yöntem

Kuvvet Serileri

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ Sağ taraflı ise z^{-n} 'li terimler

Bölme işlemi en **büyük** terimli dereceden başlanarak yapılır.

$$X(z) = \frac{1}{1-\alpha z^{-1}} \qquad |z| > |\alpha|$$

$$\begin{array}{l|l} 1 & 1-\alpha z^{-1} \\ \hline -\;1-\alpha z^{-1} & 1+\alpha z^{-1}+\alpha^2 z^{-2}+\alpha^3 z^{-3}+\cdots\infty \\ \alpha z^{-1} & \\ -\;\alpha z^{-1}-\alpha^2 z^{-2} & \\ \alpha^2 z^{-2} & \\ -\;\alpha^2 z^{-2}-\alpha^3 z^{-3} & \\ \alpha^3 z^{-3} & \\ \vdots & \end{array}$$

$$\frac{1}{1-\alpha z^{-1}} = 1+\alpha z^{-1}+\alpha^2 z^{-2}+\cdots\infty$$

$$1+\alpha^1 z^{-1}+\alpha^2 z^{-2}+\cdots\infty = \sum_{n=0}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} \alpha^n z^{-n}$$

$$x(n) = \alpha^n u(n)$$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{n}$ Sol taraflı ise z^{n} 'li terimler

Bölme işlemi en **küçük** terimli dereceden başlanarak yapılır.

$$X(z) = \frac{1}{1-\alpha z^{-1}} \qquad |z| < |\alpha|$$

$$\begin{array}{l|l} 1 & -\alpha z^{-1}+1 \\ \hline -\;1-\alpha^{-1} z^{1} & \alpha^{-1} z^{1}+\alpha^{-2} z^{2}+\alpha^{-3} z^{3}+\cdots\infty \\ \alpha^{-1} z^{1} & \\ -\;\alpha^{-1} z^{1}-\alpha^{-2} z^{2} & \\ \alpha^{-2} z^{2} & \\ -\;\alpha^{-2} z^{2}-\alpha^{-3} z^{3} & \\ \alpha^{-3} z^{3} & \\ \vdots & \end{array}$$

$$\frac{1}{1-\alpha z^{-1}} = \alpha^{-1} z^{1}+\alpha^{-2} z^{2}+\alpha^{-3} z^{3}+\cdots\infty$$

$$\alpha^{-1} z^{1}+\alpha^{-2} z^{2}+\cdots\infty = \sum_{n=-1}^{-\infty} x(n) z^{n} = \sum_{n=-1}^{-\infty} \alpha^{-n} z^{n}$$

$$x(n) = \alpha^{-n} u(-n-1)$$

Ters Z dönüşümü

3.Yöntem

Kısmi Kesirlere Ayırma Yöntemi

Örnek $\sigma(x)=\dfrac{8x^2+3x-21}{(x+2)(x-3)(x+1)}$ Ters z dönüşümünü bulunuz?

Çözüm

$$\sigma(x)=\frac{8x^2+3x-21}{(x+2)(x-3)(x+1)}=\frac{\alpha_1}{x+2}+\frac{\alpha_2}{x-3}+\frac{\alpha_3}{x+1}$$

$$\alpha_1=(x+2)\sigma(x)\big|_{x=-2}$$

$$\alpha_2=(x-3)\sigma(x)\big|_{x=3}$$

$$\alpha_3=(x+1)\sigma(x)\big|_{x=-1}$$

Örnek $\sigma(x)=\dfrac{N(x)}{(x-b)^r(x-a_1)\cdots(x-a_j)}$ **Çift Katlı kök** Ters z dönüşümünü bulunuz?

Çözüm

$$\sigma(x)=\frac{N(x)}{(x-b)^r(x-a_1)\cdots(x-a_j)}=\frac{\beta_0}{(x-b)^r}+\frac{\beta_1}{(x-b)^{r-1}}+\cdots+\frac{\beta_{r-1}}{(x-b)^1}+\frac{\alpha_1}{x-a_1}+\frac{\alpha_2}{x-a_2}+\cdots+\frac{\alpha_j}{x-a_j}$$

$$\alpha_1=(x-a)\sigma(x)\big|_{x=a_j}$$

$$\beta_0=(x-b)^r\sigma(x)\big|_{x=b}$$

$$\beta_1=\frac{d}{dx}(\beta_0)\Big|_{x=b}$$

$$\vdots$$

$$\beta_k=\frac{1}{k}\frac{d}{dx}(\beta_k-1)\Big|_{x=b}$$

Örnek $X(z)=\dfrac{z^{-1}}{1-\frac{1}{4}z^{-1}}$ $\quad |z|>\dfrac{1}{4}$ zaman domeninde tersi nedir?

Çözüm

$$X(z)=\frac{z^{-1}}{1-\frac{1}{4}z^{-1}}$$

Pay ve paydanın derecesi aynı ise bölme işlemi yapılır

$$X_1(z)=\frac{1}{1-\frac{1}{4}z^{-1}}$$

$$X(z)=\underbrace{z^{-1}}_{\text{Birim öteleme}}X_1(z)$$

$$x_1(n)=\left(\frac{1}{4}\right)^n u(n)$$

$$x(n)=x_1(n-1)$$

$$x(n)=\left(\frac{1}{4}\right)^{n-1} u(n-1)$$

Aynı işaret genlikleri aynı

$$\begin{array}{l|l} z^{-1} & 1-\frac{1}{4}z^{-1} \\ \hline -\left(z^{-1}-\frac{1}{4}z^{-2}\right) & z^{-1} \\ \hline \frac{1}{4}z^{-2} & \end{array}$$

$$X(z)=z^{-1}+\frac{\frac{1}{4}z^{-2}}{1-\frac{1}{4}z^{-1}}$$

$$x(n)=\delta(n-1)+\frac{1}{4}\left(\frac{1}{4}\right)^{n-2}u(n-2)$$

Kısmi kesirler yöntemi ile zaman domeninde tersi?

$$\begin{array}{l|l} z^{-1} & -\frac{1}{4}z^{-1}+1 \\ \hline -\left(z^{-1}-4\right) & -4 \\ \hline 4 & \end{array}$$

$$x(n)=4\delta(n)-4\left(\frac{1}{4}\right)^n u(n)$$

Örnek $X(z)=\dfrac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$ zaman domeninde tersi nedir?

Örnek $X(z)=\dfrac{4-\frac{7}{4}z^{-1}+\frac{1}{4}z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$ zaman domeninde tersi nedir?

Çözüm

$$X(z)=\frac{4-\frac{7}{4}z^{-1}+\frac{1}{4}z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

pay ve paydanın derecesi eşit olduğu için bölme işlemi yapıyoruz

$$\begin{array}{r|l} \frac{1}{4}z^{-2}-\frac{7}{4}z^{-1}+4 & \frac{1}{8}z^{-2}-\frac{3}{4}z^{-1}+1 \\ - & \hline \quad & 2 \\ \hline -\frac{1}{4}z^{-1}+2 & \end{array}$$

$$X(z)=2+\underbrace{\frac{2-\frac{1}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}}_{X_1(z)}$$

$$X_1(z)=\frac{A}{1-\frac{1}{2}z^{-1}}+\frac{B}{1-\frac{1}{4}z^{-1}}$$

$$A=\left(1-\frac{1}{2}z^{-1}\right)\frac{2-\frac{1}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}\Bigg|_{z^{-1}=2}=\frac{2-\frac{1}{2}}{1-\frac{1}{2}}=3$$

$$B=\left(1-\frac{1}{4}z^{-1}\right)\frac{2-\frac{1}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}\Bigg|_{z^{-1}=4}=\frac{2-1}{1-2}=-1$$

$$X_1(z)=\frac{3}{1-\frac{1}{2}z^{-1}}+\frac{-1}{1-\frac{1}{4}z^{-1}}$$

$$\left.\begin{matrix} |z|>\frac{1}{2} & |z|>\frac{1}{4} \\ |z|<\frac{1}{2} & |z|<\frac{1}{4} \end{matrix}\right\}$$

Yakınsama Bölgesi kombinasyonlarına bakalım $\begin{matrix}1 & 2\\ 3 & 4\end{matrix}$

1-2 $|z|>\frac{1}{2} \cap |z|>\frac{1}{4}$ $x(n)=2\delta(n)+3\left(\frac{1}{2}\right)^n u(n)-1\left(\frac{1}{4}\right)^n u(n)$ $|z|>\frac{1}{2}$

3-4 $|z|>\frac{1}{2} \cap |z|<\frac{1}{4}$ $x(n)=2\delta(n)-3\left(\frac{1}{2}\right)^n u(-n-1)+1\left(\frac{1}{4}\right)^n u(n)$ $\frac{1}{4}<|z|<\frac{1}{2}$

1-4 $|z|<\frac{1}{2} \cap |z|>\frac{1}{4}$ $x(n)=2\delta(n)-3\left(\frac{1}{2}\right)^n u(-n-1)+1\left(\frac{1}{4}\right)^n u(-n-1)$ $|z|<\frac{1}{4}$

Örnek $X(z)=\dfrac{1}{\left(1-2z^{-1}\right)\left(1-z^{-1}\right)}$ zaman domeninde tersi nedir?

Çözüm

$X(z)=\dfrac{1}{\left(1-2z^{-1}\right)\left(1-z^{-1}\right)}$ kısmi kesirler yöntemi

$$X(z)=\frac{A}{1-2z^{-1}}+\frac{B}{1-z^{-1}}$$

$$A=\left.\frac{1}{1-z^{-1}}\right|_{z^{-1}=\frac{1}{2}}=2 \qquad |z|>2 \quad |z|<2$$

$$B=\left.\frac{1}{1-2z^{-1}}\right|_{z^{-1}=1}=-1 \qquad |z|>1 \quad |z|<1$$

$$X(z)=\frac{2}{1-2z^{-1}}+\frac{-1}{1-z^{-1}}$$

$$\begin{aligned} x(n)&=2(2)^n u(n)-u(n)\\ &=\left(2^{n+1}-1\right)u(n) \end{aligned} \qquad |z|>2$$

Örnek $X(z)=\dfrac{1}{\left(1-z^{-2}\right)\left(1-z^{-1}\right)}$ zaman domeninde tersi nedir?

Çözüm

$$X(z)=\frac{1}{\underbrace{\left(1-z^{-1}\right)^2}_{\frac{A}{\left(1-z^{-1}\right)^2}+\frac{B}{\left(1-z^{-1}\right)^1}}\underbrace{\left(1+z^{-1}\right)}_{\frac{C}{1+z^{-1}}}}$$

!
işaretler

$$X(z)=\frac{A}{\left(1-z^{-1}\right)^2}+\frac{B}{\left(1-z^{-1}\right)^1}+\frac{C}{1+z^{-1}}$$

$$A=\left.\frac{1}{1+z^{-1}}\right|_{z^{-1}=1}=\frac{1}{2}$$

$$B=\left.\frac{d}{dz}(A)\right|_{z^{-1}=1}=\left.\frac{d}{dz}\left(\frac{1}{1+z^{-1}}\right)\right|_{z^{-1}=1}=\left.\frac{z^{-2}}{\left(1+z^{-1}\right)^2}\right|_{z^{-1}=1}=\frac{1}{4}$$

$$C=\left.\frac{1}{\left(1-z^{-1}\right)^2}\right|_{z^{-1}=-1}=\frac{1}{4}$$

$$X(z)=\frac{\frac{1}{2}}{\left(1-z^{-1}\right)^2}+\frac{\frac{1}{4}}{\left(1-z^{-1}\right)^1}+\frac{\frac{1}{4}}{1+z^{-1}} \qquad |z|>1 \quad |z|<1$$

$$x(n)=\frac{1}{4}(-1)^n u(n)+\frac{1}{4}u(n)+\frac{1}{2}(n+1)u(n) \qquad |z|<1$$

Formül $\quad \alpha^n u(n) \leftrightarrow \dfrac{1}{1-\alpha z^{-1}} \quad |z|>|\alpha|$

$\quad -\alpha^n u(-n-1) \leftrightarrow \dfrac{1}{1-\alpha z^{-1}} \quad |z|<|\alpha|$

$$\frac{1}{\left(1-z^{-1}\right)^2}=\underbrace{\left[-z\frac{d}{dz}\underbrace{\left(\frac{1}{1-z^{-1}}\right)}_{u(n)}\right]}_{n\cdot u(n)}\cdot z$$

$$\frac{1}{\left(1-z^{-1}\right)^2}=\frac{z^{-1}}{\left(1-z^{-1}\right)^2}\cdot z$$

$$x(n)=\frac{1}{4}(-1)^n u(n)+\frac{1}{4}u(n)+\frac{1}{2}\underbrace{(n+1)u(n)}$$

$\underbrace{(n+1)u(n+1)}_{(n+1)u(n)}$ $\left.\right\}$ $u(n+1)=0$, $n=-1$ de o olduğu için

Örnek $\quad X(z)=\dfrac{1+\frac{1}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^2} \quad |z|>\dfrac{1}{2}$ zaman domeninde tersi nedir?

Çözüm

$$X(z)=\frac{A}{\left(1-\frac{1}{2}z^{-1}\right)^2}+\frac{B}{\left(1-\frac{1}{2}z^{-1}\right)^1}$$

$$A=1+\frac{1}{4}z^{-1}\Big|_{z^{-1}=2}=\frac{3}{2}$$

$$B=\frac{d}{dz}(A)\Big|_{z^{-1}=2}=\frac{d}{dz}\left(1+\frac{1}{4}z^{-1}\right)\Big|_{z^{-1}=2}=-\frac{1}{4}z^{-2}\Big|_{z^{-1}=2}=-1$$

$$\left(1-\frac{1}{2}z^{-1}\right)^2=\left[-z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)\right]\cdot 2z$$

$$=\left[\frac{\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^2}\right]\cdot 2z$$

$$\underbrace{\underbrace{-z\frac{d}{dz}\underbrace{\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)}_{\left(\frac{1}{2}\right)^n u(n)}}_{n\cdot\left(\frac{1}{2}\right)^n u(n)}\cdot 2z}_{2\cdot(n+1)\left(\frac{1}{2}\right)^n u(n+1)}$$

$$x(n)=2\cdot(n+1)\left(\frac{1}{2}\right)^n u(n+1)$$

Örnek $X(z)=\ln\left(1-\frac{1}{2}z^{-1}\right)$ $|z|>\frac{1}{2}$ zaman domeninde tersi nedir?

Çözüm

$$-z\left(\frac{d}{dz}x(z)\right)=-z\left(\frac{-\frac{1}{2}z^{-2}}{1-\frac{1}{2}z^{-1}}\right)=\frac{-\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}=-\frac{1}{2}\cdot\frac{z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$nx(n)=-\frac{1}{2}\left(\frac{1}{2}\right)^{n-1}u(n-1)$$

$$x(n)=-\frac{1}{n}\left(\frac{1}{2}\right)^{n}u(n-1)$$

Örnek $x(n)=e^{-\alpha Tn}u(n)$ z dönüşümü nedir?

Çözüm

$$\alpha^n u(n)\leftrightarrow\frac{1}{1-\alpha z^{-1}}$$

$$\alpha=e^{-\alpha T}$$

$$X(z)=\frac{1}{1-e^{-\alpha T}z^{-1}}$$

$$|z|>e^{-\alpha T}$$

Örnek $x(n)=e^{\alpha(2n+1)}u(-n-1)+e^{-\alpha(2n+1)}u(n)$ z dönüşümü nedir?

Çözüm

$$x(n)=e^{\alpha(2n+1)}u(-n-1)+e^{-\alpha(2n+1)}u(n)$$

$$=\left(e^{2\alpha n}\cdot e^{1}\right)u(-n-1)+\left(e^{-2\alpha n}\cdot e^{-\alpha}\right)u(n)$$

$$e^{2\alpha n}=\alpha$$

$$X(z)=\frac{-e^{\alpha}}{1-e^{2\alpha}z^{-1}}+\frac{-e^{\alpha}}{1-e^{-2\alpha}z^{-1}}$$

$$|z|<e^{2\alpha} \qquad |z|<e^{-2\alpha}$$

$$e^{-2\alpha}<|z|<e^{2\alpha}$$

Örnek $x(k)+\frac{1}{3}x(k-1)=u(k)+u(k-1) \qquad k\geq 0 \quad u(k)=\left(\frac{1}{3}\right)^k$

a) İki yanlı z dönüşümü ile
b) Bir yanlı z dönüşümü ile
$k\geq 0$ için $u(k)=\left(\frac{1}{3}\right)^k$ ve $u(-1)=3$ koşulları altında
c) $k\rightarrow k+1$ ötelenmesi uygulanarak çözünüz

Soruda değişiklik yapılıp çözüm yapıldı. **Uygulama7 bkz sunu**

Çözüm

$$y(n)+\frac{1}{3}y(n-1)=x(n)+x(n-1) \qquad x(n)=\left(\frac{1}{3}\right)^n u(n) \qquad y(n)=?$$

a)

$$Y(z)+\frac{1}{3}z^{-1}Y(z)=X(z)+z^{-1}X(z)$$

$$X(z)=\frac{1}{1-\left(\frac{1}{3}\right)z^{-1}}$$

$$Y(z)\left(1+\frac{1}{3}z^{-1}\right)=X(z)\left(1+z^{-1}\right)$$

$$Y(z)\left(1+\frac{1}{3}z^{-1}\right)=\frac{1}{1-\left(\frac{1}{3}\right)z^{-1}}\left(1+z^{-1}\right)$$

$$A=\left.\frac{1+z^{-1}}{\left(1+\frac{1}{3}z^{-1}\right)}\right|_{z^{-1}=3}=2$$

$$Y(z)=\frac{1+z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}$$

$$B=\left.\frac{1+z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)}\right|_{z^{-1}=-3}=-1$$

$$Y(z)=\frac{A}{\left(1-\frac{1}{3}z^{-1}\right)}+\frac{B}{\left(1+\frac{1}{3}z^{-1}\right)}$$

$$Y(z)=\frac{2}{\left(1-\frac{1}{3}z^{-1}\right)}+\frac{-1}{\left(1+\frac{1}{3}z^{-1}\right)}$$

$$y(n)=2\left(\frac{1}{3}\right)^n u(n)-\left(-\frac{1}{3}\right)^n u(n)$$

Durum denklemlerinden transfer fonksiyonu $H(z)$

$$H(z)=\frac{Y(z)}{X(z)}$$

$$\left.\begin{aligned} q(n+1) &= A\,q(n)+B\,x(n) \\ z.Q(z) &= AQ(z)+BX(z) \end{aligned}\right\}$$

$$\underbrace{z}_{\substack{Karmaşık\\sayı}}\,.Q(z)-\underbrace{A}_{Matris}\,Q(z)=BX(z)$$

$$\cancel{(z-A)}Q(z)=BX(z)$$

Karmaşık sağı ve Matris bu şekilde ortak paranteze alınamaz o yüzden karmaşık sayı I matrisi ile çarpılır

$$(zI-A)Q(z)=BX(z)$$

$$Q(z)=(zI-A)^{-1}BX(z)$$

$$\left.\begin{aligned} y(n) &= C\,q(n)+d\;x(n) \\ Y(z) &= C\,Q(z)+d\;X(z) \end{aligned}\right\}$$

$$Y(z)=C\,Q(z)+d\;X(z)$$

$$Y(z)=C\cdot(zI-A)^{-1}BX(z)+d\;X(z)$$

$$\frac{Y(z)}{X(z)}=C\cdot(zI-A)^{-1}B+d$$

$$H(z)=C\cdot(zI-A)^{-1}B+d$$

Örnek $\quad A=\begin{bmatrix}0 & 1\\ 2 & -1\end{bmatrix} \quad B=\begin{bmatrix}0\\1\end{bmatrix} \quad C=\begin{bmatrix}3\\1\end{bmatrix} \quad d=1 \quad H(z)=?$

Çözüm

$$zI-A=\begin{bmatrix}z & -1\\ -2 & z+1\end{bmatrix}$$

$$(zI-A)^{-1}=\frac{1}{z(z+1)-2}\begin{bmatrix}z+1 & 1\\ 2 & z\end{bmatrix}$$

$$H(z)=C\cdot(zI-A)^{-1}B+d$$

$$=\begin{bmatrix}3 & 1\end{bmatrix}\left(\begin{bmatrix}z & 0\\ 0 & z\end{bmatrix}-\begin{bmatrix}0 & 1\\ 2 & -1\end{bmatrix}\right)^{-1}+1$$

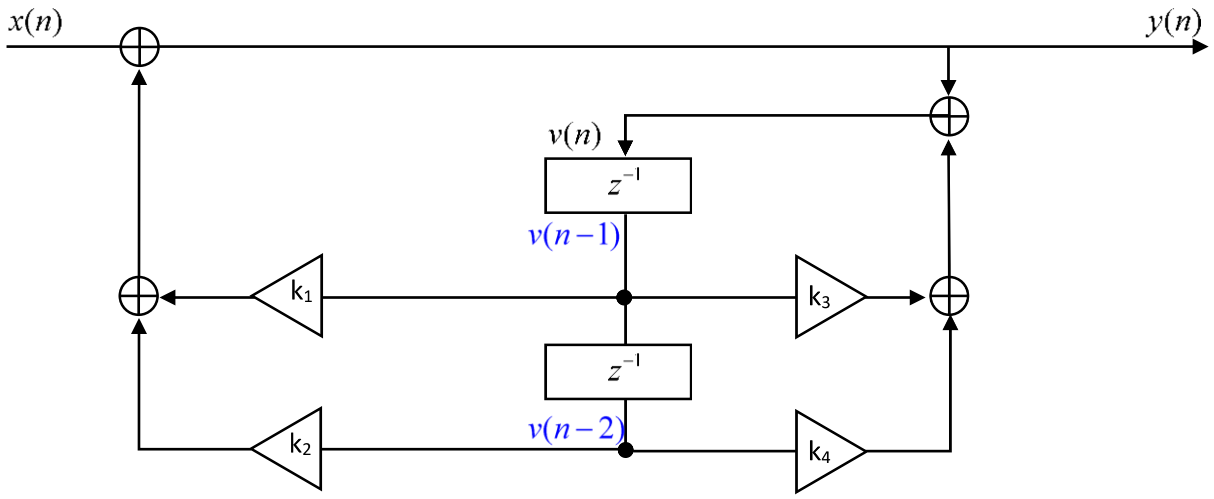
$$=\frac{3+z}{z(z+1)-2}+1$$

$$\frac{1}{z(z+1)-2}=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}z+1 & 1\\ 2 & z\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1\end{bmatrix}\begin{bmatrix}1\\z\end{bmatrix}$$

Sistemin transfer fonksiyonu

Örnek



Çözüm

$$y(n) = x(n) + \underbrace{\left(k_1 \cdot v(n-1) + k_2 \cdot v(n-2)\right)}_{1.\text{kol}} + \underbrace{\left(y(n) + k_3 \cdot v(n-1) + k_4 \cdot v(n-2)\right)}_{v(n)\ 2.\text{kol}}$$

$$v(n) = y(n) + k_3 \cdot v(n-1) + k_4 \cdot v(n-2)$$
$$V(z) = Y(z) + k_3 z^{-1} V(z) + k_4 z^{-2} V(z)$$

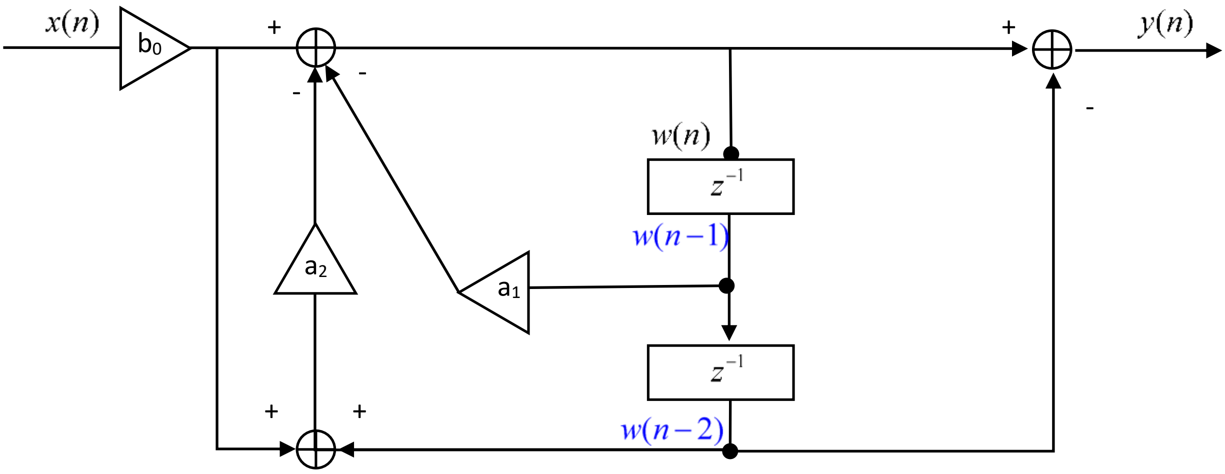
$$V(z) - k_3 z^{-1} V(z) - k_4 z^{-2} V(z) = Y(z)$$
$$V(z)\left(1 - k_3 z^{-1} - k_4 z^{-2}\right) = Y(z)$$
$$V(z) = \frac{Y(z)}{1 - k_3 z^{-1} - k_4 z^{-2}}$$

$$y(n) = x(n) + \left(k_1 \cdot v(n-1) + k_2 \cdot v(n-2)\right)$$
$$Y(z) = X(z) + k_1 z^{-1} V(z) + k_2 z^{-2} V(z)$$

$$Y(z) - k_1 z^{-1} V(z) - k_2 z^{-2} V(z) = X(z)$$
$$Y(z) - \left(k_1 z^{-1} + k_2 z^{-2}\right) V(z) = X(z)$$
$$Y(z) - \left(k_1 z^{-1} + k_2 z^{-2}\right) \frac{Y(z)}{1 - k_3 z^{-1} - k_4 z^{-2}} = X(z)$$
$$Y(z)\left(1 - \frac{k_1 z^{-1} + k_2 z^{-2}}{1 - k_3 z^{-1} - k_4 z^{-2}}\right) = X(z)$$
$$Y(z)\left(\frac{1 - k_3 z^{-1} - k_4 z^{-2} - k_1 z^{-1} - k_2 z^{-2}}{1 - k_3 z^{-1} - k_4 z^{-2}}\right) = X(z)$$
$$Y(z)\left(\frac{1 - \left(k_3 + k_1\right) z^{-1} - \left(k_4 + k_2\right) z^{-2}}{1 - k_3 z^{-1} - k_4 z^{-2}}\right) = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1 - k_3 z^{-1} - k_4 z^{-2}}{1 - \left(k_3 + k_1\right) z^{-1} - \left(k_4 + k_2\right) z^{-2}}$$

Örnek



Çözüm

$$y(n) = \omega(n) - \omega(n-2)$$
$$Y(z) = W(z) - z^{-2}W(z)$$

$$Y(z) = W(z) - z^{-2}W(z)$$
$$Y(z) = \left(1 - z^{-2}\right)W(z)$$
$$W(z) = \frac{Y(z)}{1 - z^{-2}}$$

$$\omega(n) = b_0x(n) - a_1\omega(n-1) - a_2\left(b_0x(n) + \omega(n-2)\right)$$
$$W(z) = b_0X(z) - a_1z^{-1}W(z) - a_2\left(b_0X(z) + z^{-2}W(z)\right)$$
$$W(z) = b_0X(z) - a_1z^{-1}W(z) - a_2b_0X(z) - a_2z^{-2}W(z)$$
$$W(z) + a_1z^{-1}W(z) + a_2z^{-2}W(z) = b_0X(z) - a_2b_0X(z)$$
$$W(z)\left(1 + a_1z^{-1} + a_2z^{-2}\right) = \left(b_0 - a_2b_0\right)X(z)$$
$$\frac{Y(z)}{1 - z^{-2}}\left(1 + a_1z^{-1} + a_2z^{-2}\right) = \left(b_0 - a_2b_0\right)X(z)$$
$$\frac{Y(z)}{X(z)} = \frac{\left(b_0 - a_2b_0\right)\left(1 - z^{-2}\right)}{1 + a_1z^{-1} + a_2z^{-2}}$$
$$H(z) = \frac{\left(b_0 - a_2b_0\right)\left(1 - z^{-2}\right)}{1 + a_1z^{-1} + a_2z^{-2}}$$

Örnek $h(0) = 1$

$$h(n) = \delta(n)$$
$$H(z) = 1$$

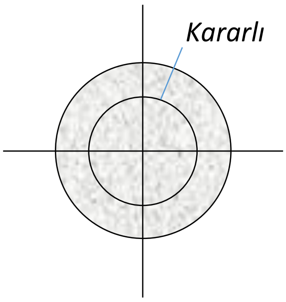
Örnek $h(0) = 2$

$$h(n) = 2\delta(n)$$
$$H(z) = 2$$

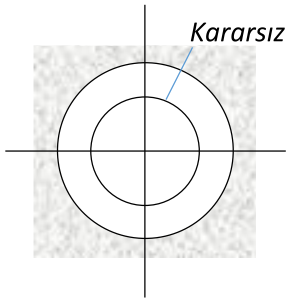
Örnek

$$H(z)=\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{2}{1-3z^{-1}}$$

Kararlı , nedensel değil $h(n)=?$



Nedensel değil
(Çemberin içi)



Nedensel
(Çemberin dışı)

Çözüm

$$\underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}\qquad \underbrace{\frac{2}{1-3z^{-1}}}$$

$|z|>\frac{1}{2}$ $|z|>3$

$|z|<\frac{1}{2}$ $|z|<3$

$$|z|>\frac{1}{2}\ \cap\ |z|<3$$

$$h(n)=\left(\frac{1}{2}\right)^n u(n)-2(3)^n u(-n-1)$$

$$\sum_{-\infty}^{+\infty}|h(n)|<\underbrace{\sum_{0}^{+\infty}\left(\frac{1}{2}\right)^n}_{\frac{1}{1-\frac{1}{2}}=2}+2\underbrace{\sum_{-\infty}^{-1}(3)^n}_{\to 0}$$

Kararsız , nedensel $h(n)=?$

$$\underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}\qquad \underbrace{\frac{2}{1-3z^{-1}}}$$

$|z|>\frac{1}{2}$ $|z|>3$

$|z|<\frac{1}{2}$ $|z|<3$

$$|z|>\frac{1}{2}\ \cap\ |z|>3$$

$$|z|>3$$

$$h(n)=\left(\frac{1}{2}\right)^n u(n)+2(3)^n u(n)$$

Kararsız , nedensel değil $h(n)=?$

$$\underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}\qquad \underbrace{\frac{2}{1-3z^{-1}}}$$

$|z|>\frac{1}{2}$ $|z|>3$

$|z|<\frac{1}{2}$ $|z|<3$

$$|z|<\frac{1}{2}\ \cap\ |z|<3$$

$$h(n)=-1\left(\frac{1}{2}\right)^n u(-n-1)-2(3)^n u(-n-1)$$

SÜREKLİ ZAMAN İŞARETLER

FOURİER SERİES

$$\omega_0 = \frac{2\pi}{T}$$

$$\phi_k(t) = e^{jk\omega_0 t} = e^{jk\left(\frac{2\pi}{T}\right)t} \qquad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

ω_0 Temel frekans katsayısı

a_k Fourier seri katsayısı

$$x(t) = \cdots + \underbrace{a_{-2}e^{-j2\omega_0 t}}_{\substack{2.Harmonik\\ kısım}} + \underbrace{a_{-1}e^{-j\omega_0 t}}_{\substack{1.Harmonik\\ kısım}} + \underbrace{a_0}_{\substack{\text{Frekansın}\\ \text{0 olduğu}\\ \text{durum}\\ \text{DC}}} + \underbrace{a_1 e^{j\omega_0 t}}_{\substack{1.Harmonik\\ kısım}} + \underbrace{a_2 e^{j2\omega_0 t}}_{\substack{2.Harmonik\\ kısım}} + \cdots$$

Örnek *Chapter3.pdf / Example 3.2*

- **Example 3.2:**

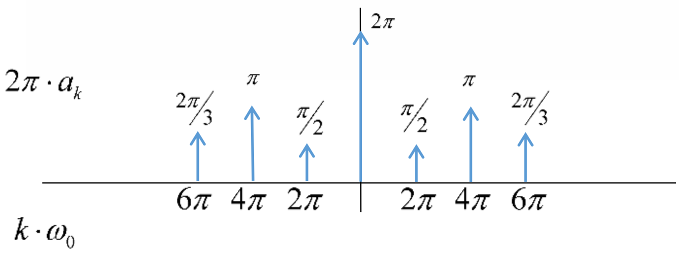
$$x(t) = \sum_{k=-3}^{+3} a_k e^{jk(2\pi)t}$$

$$a_0 = 1$$
$$a_1 = a_{-1} = \frac{1}{4}$$
$$a_2 = a_{-2} = \frac{1}{2}$$
$$a_3 = a_{-3} = \frac{1}{3}$$

$$\Rightarrow x(t) = 1 + \frac{1}{4}(e^{j2\pi t} + e^{-j2\pi t}) + \frac{1}{2}(e^{j4\pi t} + e^{-j4\pi t}) + \frac{1}{3}(e^{j6\pi t} + e^{-j6\pi t})$$

$$\Rightarrow x(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$
$$\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$
$$\sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$



$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$$

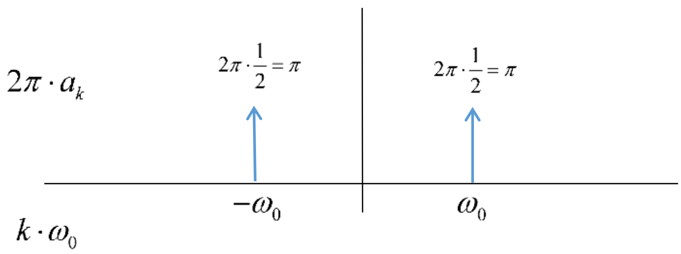
İşaretin periyodu bilinmiyorsa bu şekilde integral alınır

Örnek $x(t) = \cos(\omega_0 t)$ temel katsayısı ve frouer serisi spektrumu?

Çözüm

$$\cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$$a_1 = a_{-1} = \frac{1}{2} \qquad a_k = 0 \quad k \neq \pm 1$$

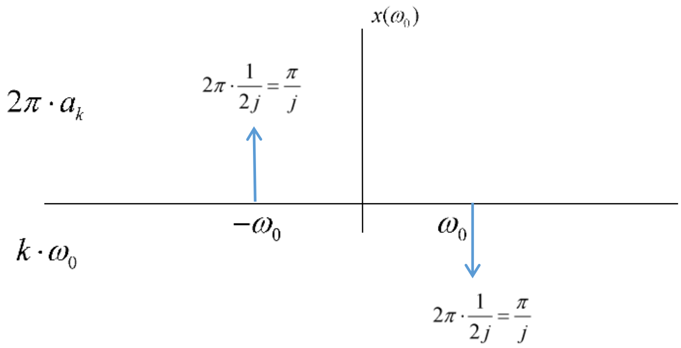


Örnek $x(t) = \sin(\omega_0 t)$ temel katsayısı ve frouer serisi spektrumu?

Çözüm

$$\sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$$

$$a_1 = \frac{1}{2j} \qquad a_{-1} = -\frac{1}{2j} \qquad a_k = 0 \quad k \neq \pm 1$$



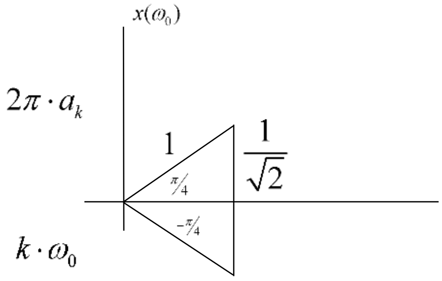
Örnek $x(t)=\cos\left(2t+\frac{\pi}{4}\right)$ temel katsayısı ve frouer işareti?

Çözüm

$$x(t)=\cos\underbrace{\left(2t+\frac{\pi}{4}\right)}_{\theta}=\frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\cos\left(2t+\frac{\pi}{4}\right)=\frac{e^{j\left(2t+\frac{\pi}{4}\right)}+e^{-j\left(2t+\frac{\pi}{4}\right)}}{2}=\frac{1}{2}\overset{j\omega_0 t}{e^{j2t}}e^{j\frac{\pi}{4}}+\frac{1}{2}e^{-j2t}e^{j\frac{\pi}{4}} \qquad \omega_0=2$$

$$a_1=\frac{e^{j\frac{\pi}{4}}}{2}=\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}} \qquad a_{-1}=\frac{e^{-j\frac{\pi}{4}}}{2}=\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$$



Örnek $x(t)=\cos(4t)+\cos(6t)$ şeklinde verilen işaretin temel katsayısı ve frouer serisi?

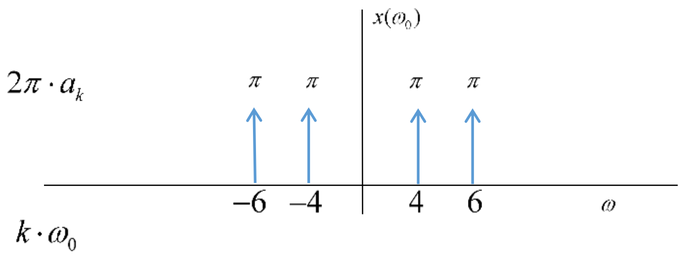
Çözüm

$$\cos\underset{\theta}{(4t)}=\frac{e^{j\theta}+e^{-j\theta}}{2} \qquad \cos\underset{\theta}{(4t)}=\frac{e^{j4t}+e^{-j4t}}{2}=\frac{1}{2}e^{j4t}+\frac{1}{2}e^{-j4t}$$

$$\cos\underset{\theta}{(6t)}=\frac{e^{j\theta}+e^{-j\theta}}{2} \qquad \cos\underset{\theta}{(6t)}=\frac{e^{j6t}+e^{-j6t}}{2}=\frac{1}{2}e^{j6t}+\frac{1}{2}e^{-j6t}$$

$$x(t)=\underbrace{\frac{1}{2}}_{a_{-3}=\frac{1}{2}}e^{-j6t}+\underbrace{\frac{1}{2}}_{a_{-2}=\frac{1}{2}}e^{-j4t}+\underbrace{\frac{1}{2}}_{a_{2}=\frac{1}{2}}e^{j4t}+\underbrace{\frac{1}{2}}_{a_{3}=\frac{1}{2}}e^{j6t}$$

$EBOB(4,6)=2$
olduğundan
$\omega_0=2$



Örnek $x(t)=\sin^2(t)$ şeklinde verilen işaretin temel katsayısı ve frouer serisi?

Çözüm

$$\sin t=\frac{e^{jt}-e^{-jt}}{2j}$$

$$\sin^2 t=\left(\frac{e^{jt}-e^{-jt}}{2j}\right)^2$$
$$=\frac{e^{j2t}}{-4}+\frac{2e^0}{4}+\frac{e^{-j2t}}{-4}$$
$$=\frac{1}{2}-\frac{1}{4}\left(e^{j2t}-e^{-j2t}\right)$$

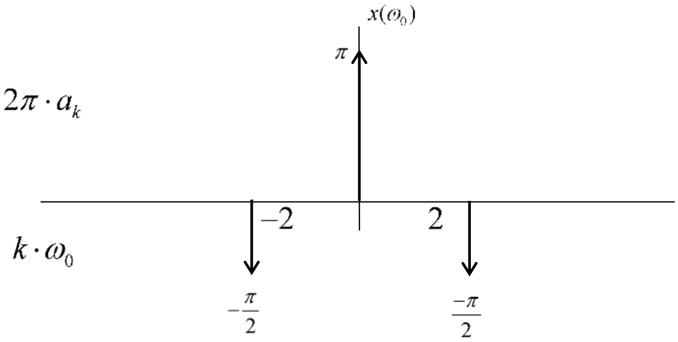
yada

$$\sin^2 t=\frac{1}{2}\left(1-\cos(2t)\right)$$
$$=\frac{1}{2}-\frac{\cos(2t)}{2}$$
$$=\frac{1}{2}-\frac{1}{4}\left(e^{j2t}-e^{-j2t}\right)$$

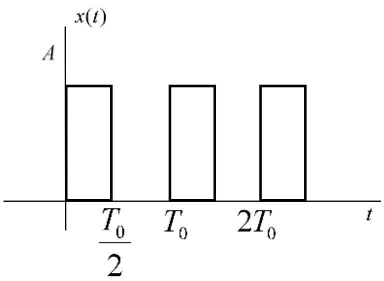
$j^2=-1$

$\omega_0=2$

$a_1=-\frac{1}{4}\qquad a_{-1}=-\frac{1}{4}\qquad a_0=\frac{1}{2}$



Örnek *Yandaki şekle göre* $a_k=?$



Çözüm $\omega_0=\frac{2\pi}{T_0}$

$$a_k=\frac{1}{T_0}\int_0^{T_0/2}x(t)e^{-jk\omega_0 t}dt$$
$$=\frac{1}{T_0}\int_0^{T_0/2}Ae^{-jk\omega_0 t}dt$$
$$=\frac{1}{T_0}\cdot A\cdot\frac{1}{jk\omega_0}$$
$$=\frac{1}{T_0}\cdot A\cdot\frac{1}{jk\left(\frac{2\pi}{T_0}\right)}\left(e^{-jk\frac{2\pi}{T_0}\frac{T_0}{2}}-1\right)$$
$$=\frac{A}{j2\pi k}\left(1-e^{-j\pi k}\right)$$
$$=\frac{A}{j2\pi k}\left(1-(-1)^k\right)$$

$$e^{-j\pi}=-1$$
$$e^{-j\pi}=(-1)^k$$

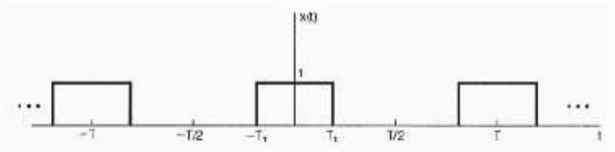
$$a_k=\frac{A}{j2\pi k}\left(1-(-1)^k\right)\quad\begin{cases}0 & k \text{ çift}\\ \frac{A}{j\pi k} & k \text{ tek}\end{cases}$$

$$a_0=\frac{A}{j2\pi 0}\left(1-(-1)^0\right)=\frac{0}{0}\ \textit{belirsizliği}$$

$$a_0=\frac{1}{T_0}\int_0^{T_0/2}A\,dt=\frac{A}{T_0}t\Big|_0^{T_0/2}=\frac{A}{T_0}\left(\frac{T_0}{2}-0\right)=\frac{A}{2}$$

Örnek *Chapter3.pdf / Example 3.2*

- **Example 3.5:** $a_k = \frac{1}{T}\int_T x(t)e^{-jkw_0t}dt = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$



$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} dt = \frac{2T_1}{T}$$

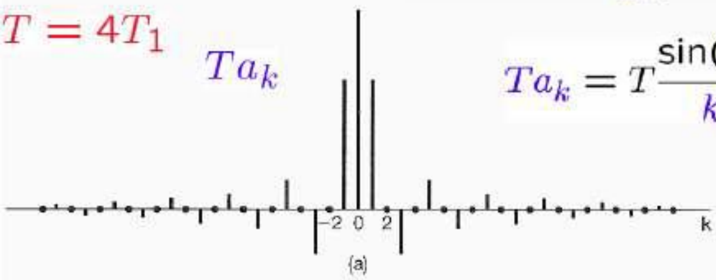
$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jkw_0t}dt = -\frac{1}{jkw_0T}e^{-jkw_0t}\Big|_{-T_1}^{T_1}$$

$$= \frac{2}{kw_0T}\left[\frac{e^{jkw_0T_1} - e^{-jkw_0T_1}}{2j}\right] \qquad w_0 = \frac{2\pi}{T}$$

$$= \frac{2\sin(kw_0T_1)}{kw_0T} = \frac{\sin(kw_0T_1)}{k\pi} = \frac{\sin(k(2\pi/T)T_1)}{k\pi}, \quad k \neq 0$$

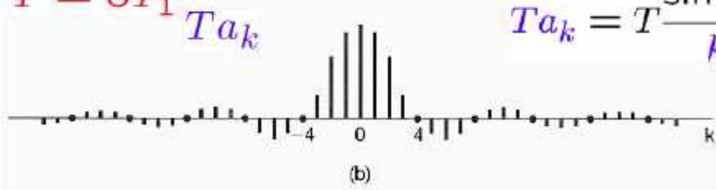
- **Example 3.5:** $Ta_k = T\frac{\sin(k2\pi\frac{T_1}{T})}{k\pi}$ $\qquad w_0 = \frac{2\pi}{T}$

$T = 4T_1$ $\qquad Ta_k$ $\qquad Ta_k = T\frac{\sin(k\frac{\pi}{2})}{k\pi}$



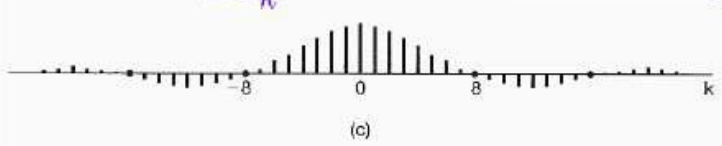
(a)

$T = 8T_1$ $\qquad Ta_k$ $\qquad Ta_k = T\frac{\sin(k\frac{\pi}{4})}{k\pi}$



(b)

$T = 16T_1$ $\qquad Ta_k$ $\qquad Ta_k = T\frac{\sin(k\frac{\pi}{8})}{k\pi}$



(c)

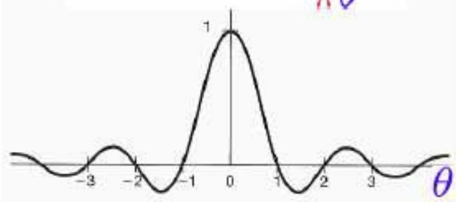
$$w = kw_0$$

$$Ta_k = \frac{2\sin(wT_1)}{w}$$

$$wT_1 = k\left(\frac{2\pi}{T}\right)\cdot T_1$$

$$= \frac{2k\pi}{A}$$

$$\text{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}$$



$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$$

$$a_1 = \frac{\sin(\omega_0 T_1)}{\pi} \qquad a_{-1} = \frac{\sin(\omega_0 T_1)}{\pi}$$

$$a_2 = \frac{\sin(2\omega_0 T_1)}{2\pi} \qquad a_{-2} = \frac{\sin(2\omega_0 T_1)}{2\pi}$$

$a_0 = \frac{0}{0}$ belirsizliği $L'Hospital$ (k ya göre) uygulanır

$$LHospital \quad \frac{\sin(k\omega_0 T_1)}{k\pi} = \frac{\omega_0 T_1 \cos(k\omega_0 T_1)}{\pi} = \frac{\omega_0 T_1}{\pi} = \frac{2\pi}{T}\frac{T_1}{\pi} = \frac{2T_1}{T}$$

$$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} dt$$

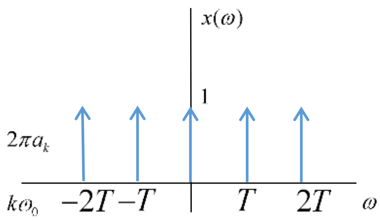
Örnek $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$

Çözüm

$k = -2 \; ise$

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$= \cdots + \delta(t+2T) + \delta(t+T) + \delta(t) + \delta(t-T) + \delta(t-2T) + \cdots$$



$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jkw_0 t} dt$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} \underset{0}{\delta(t)}\, \underset{1}{e^{-jkw_0 0}}\, dt = \frac{1}{T}$$

$$\omega_0 = \frac{2\pi}{T}$$

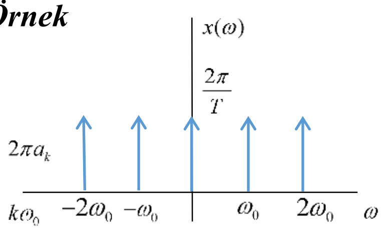
$$e^{-j2\pi} = +1$$

$\delta(t)$ *yerine* $\delta(t-T)$ *seçilseydi;*

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jkw_0 t} dt$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} \underset{0}{\delta(t-T)}\, \underbrace{e^{-jk\left(\frac{2\pi}{T}\right)T}}_{(-1)^k}\, dt = \frac{1}{T}$$

Örnek



$$x(t) = \underbrace{\frac{2\pi}{T}}_{\text{Genlik}} \sum_{k=-\infty}^{\infty} \delta\underbrace{\left(\omega - k\omega_0\right)}_{\text{burayı 0 yapan değer}}$$

Örnek $$x(t) = \sum_{k=-\infty}^{\infty} \delta\left(t - kT\right) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jkw_0 t} = \frac{1}{T} \sum_{k} e^{jkw_0 t}$$

Ödev $$x(t) = \left(1 + \cos\left(2\pi t\right)\right)\sin\left(10\pi t + \frac{\pi}{6}\right)$$

$$x(t) \to a_k \qquad x(t) = \sum_k a_k e^{jkw_0 t}$$

$$y(t) \to b_k \qquad y(t) = \sum_k b_k e^{jkw_0 t}$$

$$z(t) = Ax(t) + By(t) \qquad c_k = Aa_k + Bb_k \qquad z(t) = \sum_k c_k e^{jkw_0 t}$$

Sabit
DC

FOURİER SERİES

Öteleme

$$x(t-t_0) \leftrightarrow e^{jkw_0 t_0} a_k$$

$$x(-t) \leftrightarrow a_{-k}$$

$$z(t) = x(t)y(t) \to \sum x(t)y(t-1)$$

$$\frac{dx(t)}{dt} \leftrightarrow jk\omega_0 a_k$$

$$\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{jk\omega_0} a_k$$

$$a_k = \begin{cases} e^{-jkw_0} \sin\left(k\dfrac{\pi}{2}\right) & k \neq 0 \\ \dfrac{1}{2} & k = 0 \end{cases}$$

$$x(t-t_0) \leftrightarrow e^{jkw_0 t_0} a_k$$

$$g(t) = x(t-1) - \frac{1}{2}$$

$$x(t-1) = b_k$$

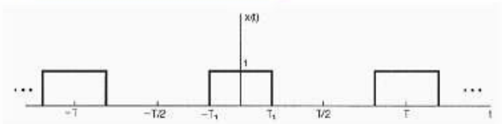
$$b_k = e^{jkw_0} a_k$$

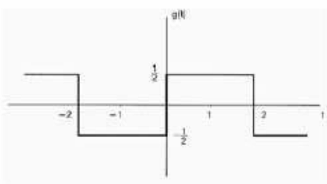
$$g(t) \leftrightarrow c_k = \begin{cases} b_k & k \neq 0 \\ 0 & k = 0 \end{cases}$$

$$x(t-1) = \sum b_k \cdot e^{jkw_0} a_k$$

Örnek *Chapter3.pdf / Example 3.6*

- Example 3.6:





$$x(t) \overset{\mathcal{FS}}{\longleftrightarrow} a_k$$

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

$$a_0 = \frac{2T_1}{T}$$

$$a_k = \frac{\sin(k(2\pi/T)T_1)}{k\pi}, \quad k \neq 0$$

$$g(t) = x(t-1) - 1/2$$

with $T = 4, T_1 = 1$

$$x(t-1) \overset{\mathcal{FS}}{\longleftrightarrow} b_k = a_k e^{-jk\pi/2}$$

$$g(t) = x(t-1) - 1/2 \overset{\mathcal{FS}}{\longleftrightarrow} \begin{cases} a_k e^{-jk\pi/2}, & \text{for } k \neq 0 \\ a_0 - 1/2, & \text{for } k = 0 \end{cases}$$

$$g(t) \overset{\mathcal{FS}}{\longleftrightarrow} \begin{cases} \dfrac{\sin(k\pi/2)}{k\pi} e^{-jk\pi/2}, & \text{for } k \neq 0 \\ 0, & \text{for } k = 0 \end{cases}$$

$$g(t) = x(t-1) - \frac{1}{2}$$
$$= \sum b_k e^{jk\omega_0 t}$$
$$= \cdots + b_{-2}e^{-j2\omega_0 t} + b_{-1}e^{-j\omega_0 t} + \underbrace{b_0}_{sabit} + b_1 e^{j\omega_0 t} + b_2 e^{j2\omega_0 t} + \cdots \underbrace{-\frac{1}{2}}_{sabit}$$

$g(t)$ DC bileşeni

$$g(t) = x(t-1) - \frac{1}{2}e^{j\omega_0 t}$$
$$= \cdots + b_{-2}e^{-j2\omega_0 t} + b_{-1}e^{-j\omega_0 t} + b_0 + \underline{b_1 e^{j\omega_0 t}} + b_2 e^{j2\omega_0 t} + \cdots \underline{-\frac{1}{2}e^{j\omega_0 t}}$$

Bu durumda

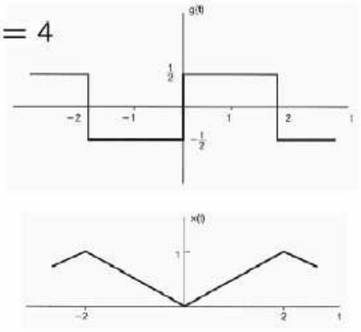
b1'e etki eder

amaç c'leri bulmak

Örnek *Chapter3.pdf / Example 3.7*

- Example 3.7:

$T = 4$



$$g(t) \overset{\mathcal{FS}}{\longleftrightarrow} d_k$$
$$x(t) \overset{\mathcal{FS}}{\longleftrightarrow} e_k$$
$$\frac{d}{dt}x(t) \overset{\mathcal{FS}}{\longleftrightarrow} jkw_0\, e_k$$

Türev alınarak kare dalga üçgen dalgaya dönüştürülür

$$g(t) = \frac{d}{dt}x(t) \iff d_k = jk(\pi/2)e_k$$

$$e_k = \begin{cases} \frac{2}{jk\pi}\, d_k = \frac{2\sin(\pi k/2)}{j(k\pi)^2}e^{-jk\pi/2}, & \text{for } k \neq 0 \\ \frac{1}{2}, & \text{for } k = 0 \end{cases}$$

$$g(t) = \frac{d}{dt}y(t) \Leftrightarrow c_k = jk\omega_0 dk$$
$$d_k = \frac{c_k}{jk\omega_0}$$
$$d_k = \begin{cases} \frac{2}{jk\pi}dk = \frac{2\sin(\pi k/2)}{j(k\pi)^2}e^{-jk\pi/2} & k \neq 0 \\ \frac{1}{2} & k = 0 \end{cases}$$

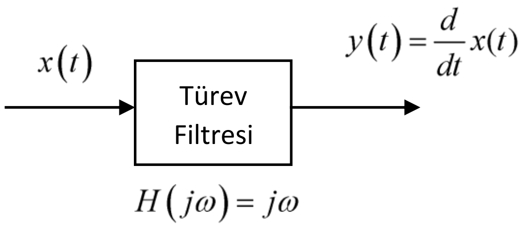
Seçkin hoca tahtada bu soruyu çözerken

d_k yerine c_k, e_k yerine d_k kullandı

amaç d'leri bulmak

FOURİER SERİES

Türev Filtresi



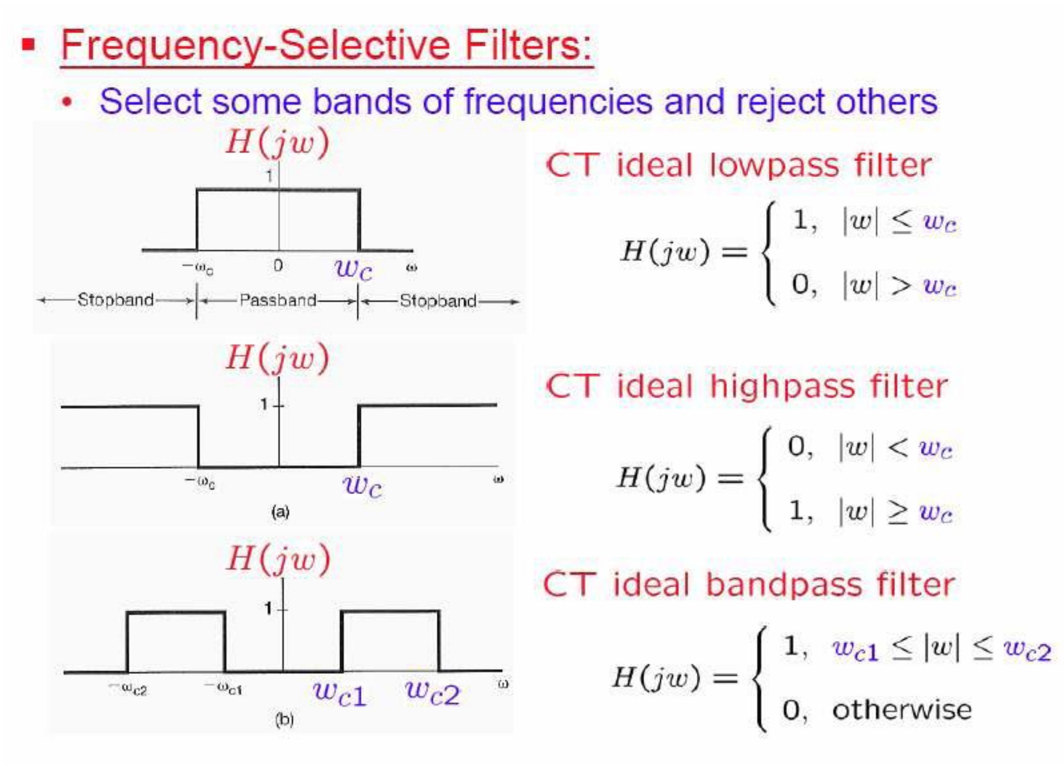
Yüksek frekanslı olanların daha baskın

Düşük frekanslı olanların 0'a yakın

$y(n) = x(n) * h(n)$ $\qquad$ $y(t) = x(t) * h(t)$

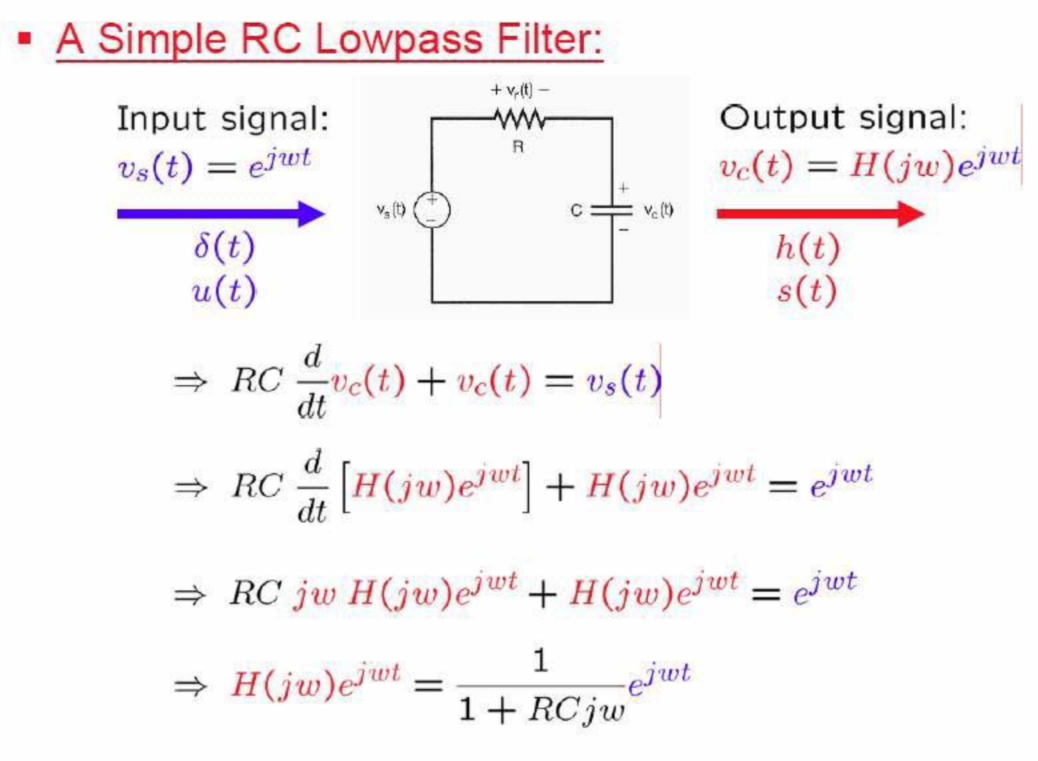
$Y(z) = X(z).H(z)$ $\qquad$ $Y(\omega) = X(\omega).H(\omega)$

1. **Alçak bant geçiren filtre**
2. **İdeal yüksek geçiren filtre**
3. **Band geçiren filtre**



FOURİER SERİES

Alçak geçiren filtre



$$V_s(t) = R.i + V_c(t)$$

$$i = C\frac{dV_c(t)}{dx}$$

$$V_s(t) = R.C\frac{dV_c(t)}{dx} + V_c(t)$$

$$V_s(t) = e^{j\omega t}$$

$$H(j\omega) = \frac{V_c(\omega)}{V_s(\omega)}$$

$$V_s(t) = R.C\frac{d}{dt}V_c(t) + V_c(t)$$

$$e^{j\omega t} = R.C\frac{d}{dt}\left[H(j\omega)e^{j\omega t}\right] + H(j\omega)e^{j\omega t}$$

$$e^{j\omega t} = R.C\; j\omega H(j\omega)e^{j\omega t} + H(j\omega)e^{j\omega t}$$

$$H(j\omega)e^{j\omega t} = \frac{1}{1 + RCj\omega}e^{j\omega t}$$

$$h(t) = \frac{1}{RC}e^{-t/RC}u(t)$$

İşaretlerimiz periyodik değilse

FOURİER SERİES dönüşümü

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} x(j\omega)e^{-j\omega t}d\omega$$

$$x(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}d\omega$$

$$a_k = \frac{1}{T}x(j\omega)\Big|_{\omega = k\omega_0}$$

$$X(z) = \sum x(n)z^{-n}$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-jk\omega t}dt$$

Fourier dönüşümü

Zamana göre

Frekansa göre

$$X(t) = \frac{1}{2\pi}\int X(\omega)e^{jk\omega t}d\omega$$

Fourier ters dönüşümü

Örnek $x(t)=e^{-at}u(t)$

Çözüm Periyodik olmadığı için fourier dönüşümü yapılır

$$x(t)=e^{-at}u(t)$$

$$X(\omega)=\int_{0}^{\infty} e^{-at}e^{-j\omega t}$$

$$=\int_{0}^{\infty} e^{-(a+j\omega)t}$$

$$=\frac{-1}{a+j\omega}e^{-(a+j\omega)t}\Big|_{0}^{\infty}$$

$$=-\frac{1}{a+j\omega}(0-1)$$

$$=\frac{1}{a+j\omega}$$

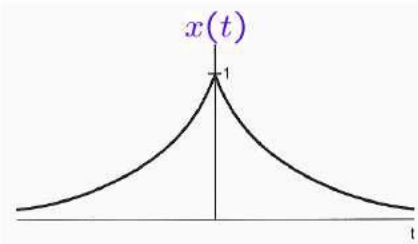
Alçak geçiren şeklinde davranan spektrum

Örnek *Chapter4.pdf / Example 4.2*

- Example 4.2:

$$X(jw)=\int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$x(t)=e^{-a|t|}, \quad a>0$$

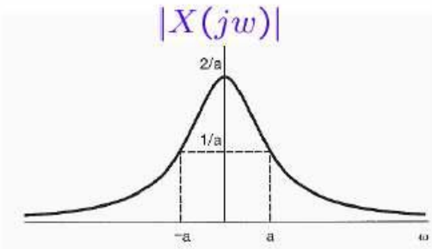


$$\Rightarrow X(jw)=\int_{-\infty}^{\infty} e^{-a|t|}\, e^{-jwt}dt$$

$$=\int_{-\infty}^{0} e^{at}\, e^{-jwt}dt \quad + \int_{0}^{\infty} e^{-at}\, e^{-jwt}dt$$

$$=\frac{1}{a-jw}+\frac{1}{a+jw}$$

$$=\frac{2a}{a^2+w^2}$$

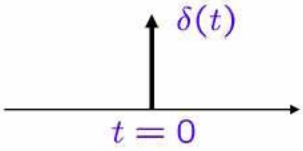


Örnek *Chapter4.pdf / Example 4.3*

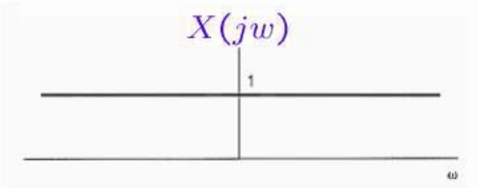
- Example 4.3:

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$$

$x(t) = \delta(t)$, i.e., unit impules



$$\Rightarrow X(jw) = \int_{-\infty}^{\infty} \delta(t)\, e^{-jwt} dt = 1$$

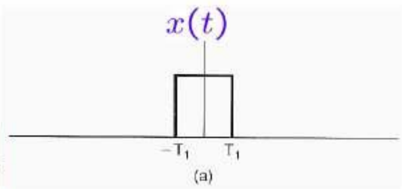


Örnek *Chapter4.pdf / Example 4.4*

- Example 4.4:

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$$

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$



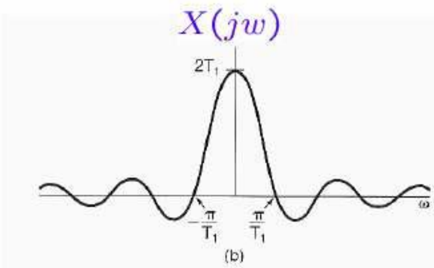
$$\Rightarrow X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-T_1}^{T_1} e^{-jwt} dt$$

$$= \frac{1}{-jw} e^{-jwt} \Big|_{-T_1}^{T_1}$$

$$= \frac{1}{-jw}\left(e^{-jwT_1} - e^{jwT_1}\right)$$

$$= \frac{1}{jw}\left(e^{jwT_1} - e^{-jwT_1}\right)$$

$$= 2\frac{\sin(wT_1)}{w}$$



Örnek *Chapter4.pdf / Example 4.5*

- Example 4.5:

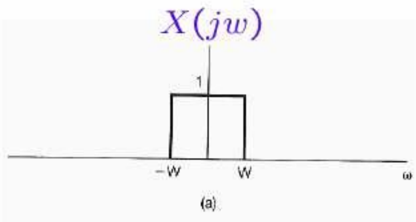
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(jw)e^{jwt}dw$$

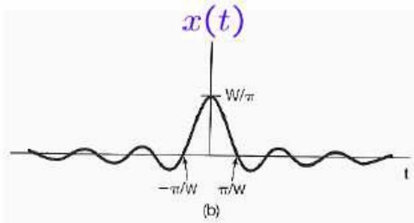
$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$X(jw) = \begin{cases} 1, & |w| < W \\ 0, & |w| > W \end{cases}$$

$$\Rightarrow x(t) = \frac{1}{2\pi}\int_{-W}^{W} e^{jwt}dw$$

$$= \frac{\sin(Wt)}{\pi t}$$

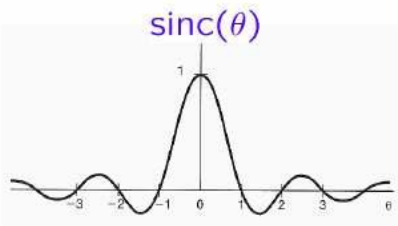


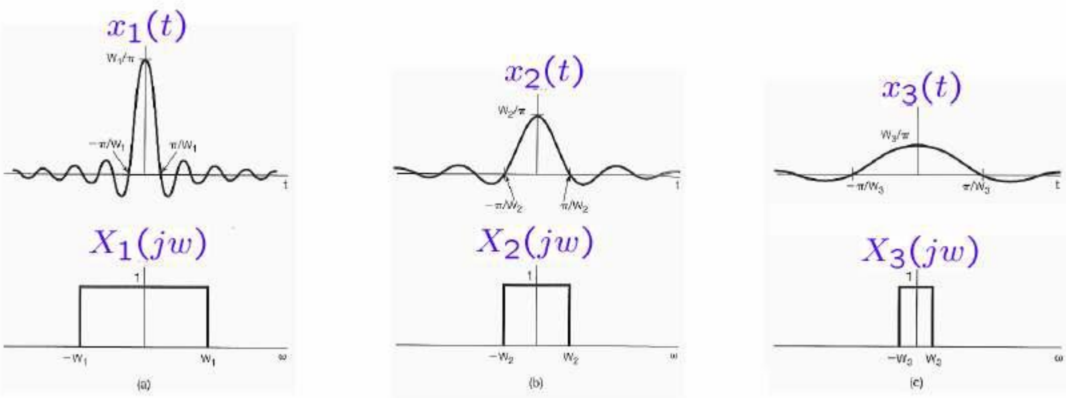


- sinc functions:

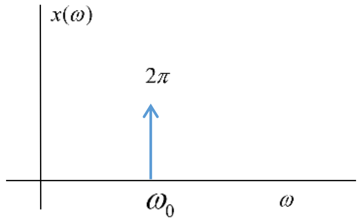
$$\mathrm{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}$$

$$\frac{\sin(Wt)}{\pi t} = \frac{W}{\pi}\mathrm{sinc}\left(\frac{Wt}{\pi}\right)$$





Örnek $x(t)=?$



Çözüm

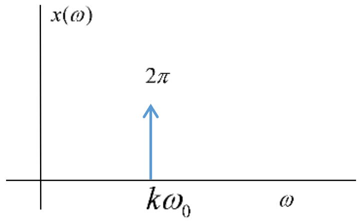
$$X(\omega)=2\pi\delta\left(\omega-\omega_0\right)$$

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X\left(\omega\right)e^{j\omega t}d\omega$$

$$=\frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty} \cancel{2\pi}\delta\underbrace{\left(\omega-\omega_0\right)}_{0 \text{ olması için } \omega=\omega_0} e^{j\omega t}d\omega$$

$$=e^{j\omega_0 t}$$

Örnek $x(t)=?$



Çözüm

$$X(\omega)=2\pi\delta\left(\omega-k\omega_0\right)\Rightarrow e^{jk\omega_0 t}$$

$$x(t)=\sum a_k e^{jk\omega_0 t}$$

$$\boldsymbol{F}\left(x(t)\right)=\boldsymbol{F}(\sum a_k e^{jk\omega_0 t})$$

$$\boldsymbol{F}\left(e^{jk\omega_0 t}\right)=\boldsymbol{F}(\sum a_k e^{jk\omega_0 t})$$

$$F\left(e^{jk\omega_0 t}\right)=2\pi\delta\left(\omega-k\omega_0\right)$$

$$\sum \boldsymbol{F}\left(a_k e^{jk\omega_0 t}\right)=\sum a_k \boldsymbol{F}\left(e^{jk\omega_0 t}\right)$$

$$X(\omega)=\sum \underbrace{a_k}_{\text{Genlik}} 2\pi\,\delta\left(\omega-k\omega_0\right)$$

FOURIER SERİSİ AÇILIMI (Özet-Tekrar)

$$x(t) = \sum a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Örnek

$$\begin{aligned} x(t) &= \sum a_k e^{jk\omega_0 t} \\ F(x(t)) &= F\left(\sum a_k e^{jk\omega_0 t}\right) \\ &= \sum F\left(a_k e^{jk\omega_0 t}\right) \\ &= \sum a_k F\left(e^{jk\omega_0 t}\right) \end{aligned}$$

$$F\left(e^{jk\omega_0 t}\right) = 2\pi\delta(\omega - k\omega_0)$$

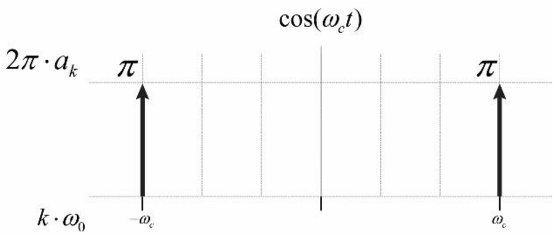
$$X(\omega) = \sum a_k F\left(e^{jk\omega_0 t}\right)$$

$$X(\omega) = \sum a_k 2\pi\delta(\omega - k\omega_0)$$

Örnek $\cos(\omega_c t)$ spektrumu

$$\begin{aligned} \cos(\omega_c t) &= \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} \\ &= \underbrace{\frac{1}{2}}_{a_{-1}} e^{-j\omega_c t} + \underbrace{\frac{1}{2}}_{a_1} e^{j\omega_c t} \end{aligned}$$

$$a_1 = a_{-1} = \frac{1}{2} \qquad a_0 = 0 \quad k \neq \pm 1$$



FOURİER SERİES

DOĞRUSALLIK

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}dt$$

$$X(j\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$$

$$x(t)\xrightarrow{F}X(j\omega)$$

$$y(t)\xrightarrow{F}Y(j\omega)$$

$$ax(t)+by(t)\longleftrightarrow aX(j\omega)+bY(j\omega)$$

$$F\left(ax(t)+by(t)\right)\xleftrightarrow{F}F\left(aX(j\omega)\right)+F\left(bY(j\omega)\right)$$

$$F\left(ax(t)+by(t)\right)\xleftrightarrow{F}aF\left(X(j\omega)\right)+bF\left(Y(j\omega)\right)$$

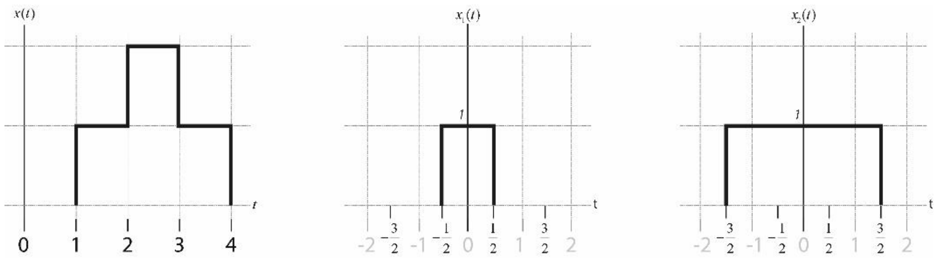
FOURİER SERİES

ÖTELEME

$x(t)$

$$x(t-t_0)\longleftrightarrow e^{-j\omega t_0}X(\omega)$$

Örnek $X(\omega)=\frac{2}{\omega}\sin(\omega T_1)$



$$x(t)=\frac{1}{2}x_1(t-2,5)+x_2(t-2,5)$$

$$X_1(j\omega)=\frac{2\sin\left(\omega/2\right)}{\omega}=\frac{2}{\omega}\sin\left(\omega/2\right) \qquad X_2(j\omega)=\frac{2\sin\left(3\omega/2\right)}{\omega}=\frac{2}{\omega}\sin\left(3\omega/2\right)$$

$$x_1(t)=e^{-j\omega t_0}X_1(\omega)=e^{-j\omega\frac{5}{2}}\frac{2}{\omega}\sin\left(\frac{\omega}{2}\right) \qquad x_2(t)=e^{-j\omega t_0}X_2(\omega)=e^{-j\omega\frac{5}{2}}\frac{2}{\omega}\sin\left(\frac{3\omega}{2}\right)$$

$$x(t)=\frac{1}{2}x_1(t-\frac{5}{2})+x_2(t-\frac{5}{2})$$

$$X(\omega)=\frac{1}{2}e^{-j\omega\frac{5}{2}}\frac{2}{\omega}\sin\left(\frac{\omega}{2}\right)+e^{-j\omega\frac{5}{2}}\frac{2}{\omega}\sin\left(\frac{3\omega}{2}\right)$$

$$=e^{-j\omega\frac{5}{2}}\left(\frac{\sin\left(\frac{\omega}{2}\right)+2\sin\left(\frac{3\omega}{2}\right)}{\omega}\right)$$

FOURİER SERİES

ÇİFT TARAFLI ÖZELLİĞİ (DUALITY)

$$x(t) \longleftrightarrow X(\omega)$$

$$X(t) \longleftrightarrow 2\pi x(-\omega)$$

Örnek $\dfrac{\sin(at)}{\pi t}$ işaretinin spektrumu (*Chapter4.pdf / Example 4.4*)

$$x(t) = \begin{cases} 1 & |t| < |a| \\ 0 & diğer \end{cases}$$

$$\frac{2}{t}\sin(at) \longleftrightarrow 2\pi \begin{cases} 1 & |t| < |a| \\ 0 & diğer \end{cases}$$

$$\frac{1}{2\pi}\frac{2}{t}\sin(at) \longleftrightarrow \frac{1}{2\pi}2\pi \begin{cases} 1 & |t| < |a| \\ 0 & diğer \end{cases}$$

$\dfrac{\sin(at)}{\pi t}$ ifadesine benzetmeye çalışıyoruz

Örnek $x(t) = e^{-a|t|}$ $a > 0$ işaretinin spektrumu (*Chapter4.pdf / Example 4.2*)

$$x(t) = e^{-a|t|} \longleftrightarrow \frac{2a}{a^2 + \omega^2}$$

Örnek $\dfrac{1}{a^2 + t^2}$ işaretinin spektrumu (bkz. *Chapter4.pdf / Example 4.2*)

$$e^{-a|t|} \longleftrightarrow \frac{2a}{a^2 + \omega^2}$$

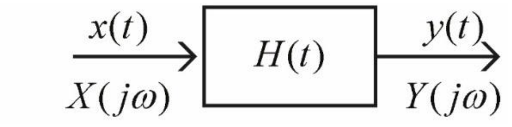
$$\frac{1}{2a}\frac{2a}{a^2 + t^2} \longleftrightarrow \frac{1}{2a}2\pi e^{-a|\omega|}$$

$$\frac{1}{a^2 + t^2} \longleftrightarrow \frac{\pi}{a}e^{-a|\omega|}$$

$\dfrac{1}{a^2 + t^2}$ ifadesine benzetmeye çalışıyoruz

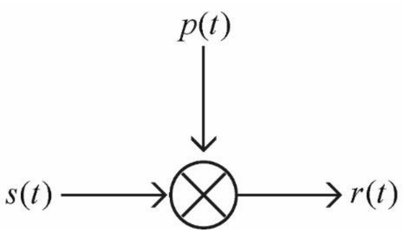
FOURİER SERİES

KONVOLÜSYON



Ayrık Zamanlı Sistem

$$y(t) = x(t) * h(t)$$
$$Y(\omega) = X(\omega) \cdot H(\omega)$$

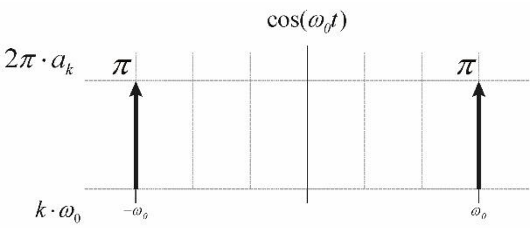


Sürekli Zamanlı Sistem

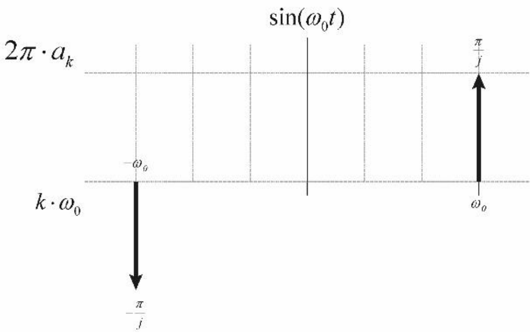
$$r(t) = s(t) * p(t)$$
$$R(\omega) = \frac{1}{2\pi}\left[S(\omega) \cdot P(\omega)\right]$$

Örnek $\cos(\omega_0 t) \cdot \sin(\omega_0 t)$ işaretinin spektrumu nedir?

Çözüm $\underbrace{\cos(\omega_0 t)}_{x_1(t)} \cdot \underbrace{\sin(\omega_0 t)}_{x_2(t)}$ $\quad X(\omega) = \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]$



$$X_1(\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$



$$X_2(\omega) = \frac{\pi}{j}\delta(\omega - \omega_0) - \frac{\pi}{j}\delta(\omega + \omega_0)$$

$$X(\omega) = \left(\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)\right) * X_2(\omega)$$
$$= \pi X_2(\omega - \omega_0) + \pi X_2(\omega + \omega_0)$$

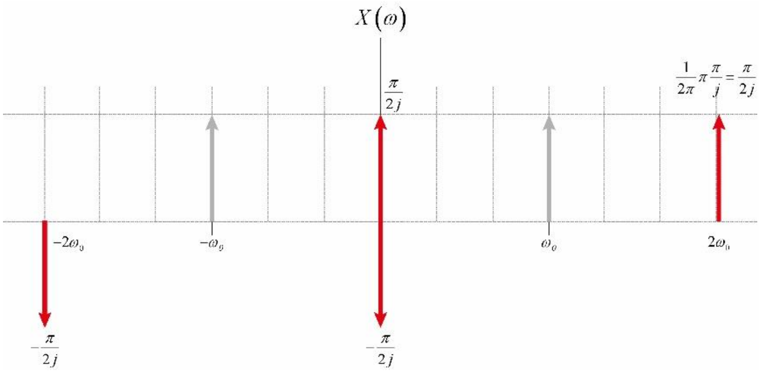
veya

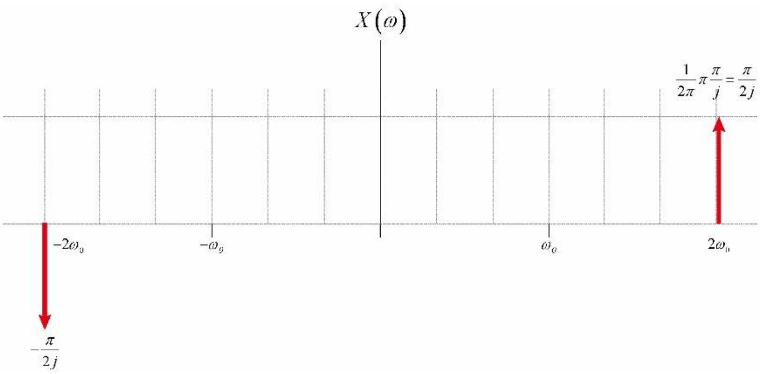
$$X(\omega) = X_1(\omega) * \left(\frac{\pi}{j}\delta(\omega - \omega_0) - \frac{\pi}{j}\delta(\omega + \omega_0)\right)$$
$$= \frac{\pi}{j} X_1(\omega - \omega_0) - \frac{\pi}{j} X_1(\omega + \omega_0)$$

$$x(n) * \delta(n-2) = x(n-2)$$

$$x_1(n) = \delta(n-1)$$
$$x_2(n) = \delta(n-2)$$

$$x_1(n) * x_2(n) = x_1(n) * \delta(n-2)$$
$$= x_2(n) * \delta(n-1)$$

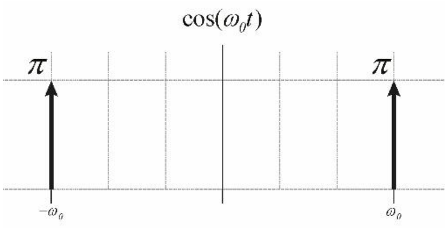




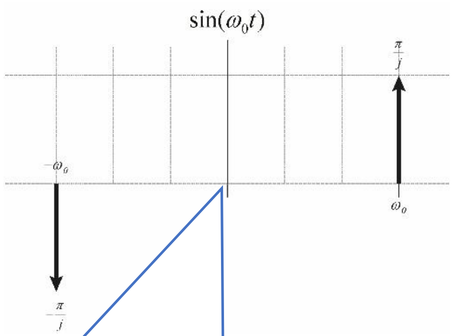
$$x(t) = \frac{1}{2}\sin(2\omega_0)$$

2.Yol

$$\frac{1}{2\pi}\Bigg[$$



*



$$\Bigg]$$

Bir işaretin 0 noktasından tutup diğer işaretin bulunduğu tüm noktalara yerleştirme işlemi yapıyoruz. Üst üste gelen işaretler toplanıyor.

Örnek *Chapter4.pdf / Example 4.20*

Example 4.20:

$$H(jw) = \int_{-\infty}^{\infty} h(t)e^{-jwt}dt$$

$$x(t) = \frac{\sin(w_i t)}{\pi t} \rightarrow \text{Filter LTI System} \rightarrow y(t) = ?$$

$$h(t) = \frac{\sin(w_c t)}{\pi t}$$

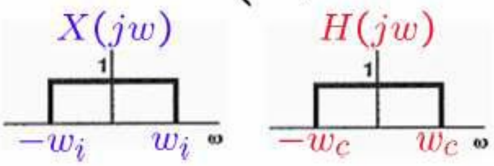
$$\Rightarrow Y(jw) = H(jw)X(jw)$$

$$\Rightarrow X(jw) = \begin{cases} 1, & |w| \le w_i \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow H(jw) = \begin{cases} 1, & |w| \le w_c \\ 0, & \text{otherwise} \end{cases}$$

$$w_0 = \min(w_c, w_i)$$

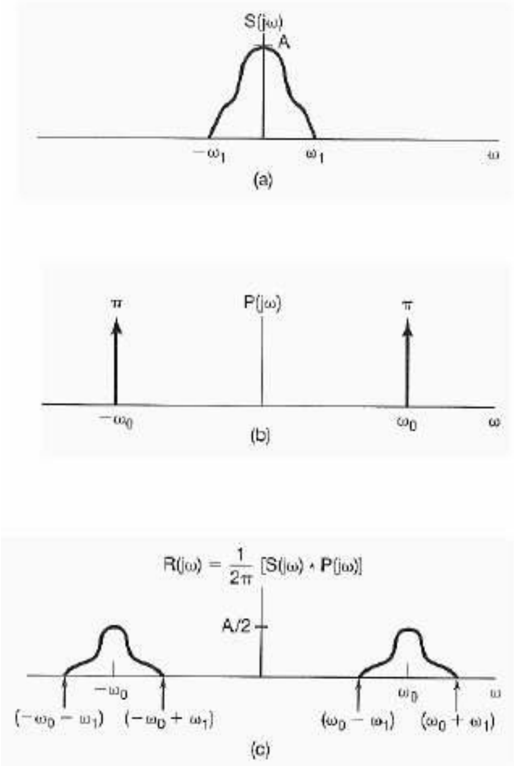
$$\Rightarrow Y(jw) = \begin{cases} 1, & |w| \le w_0 \\ 0, & \text{otherwise} \end{cases}$$



$$\Rightarrow y(t) = \begin{cases} \frac{\sin(w_c t)}{\pi t}, & w_c \le w_i \\ \frac{\sin(w_i t)}{\pi t}, & w_c \ge w_i \end{cases}$$

Örnek *Chapter4.pdf / Example 4.21*

Example 4.21:



$$r(t) = s(t)p(t)$$

$$s(t) \overset{\mathcal{F}}{\longleftrightarrow} S(jw)$$

$$p(t) \overset{\mathcal{F}}{\longleftrightarrow} P(jw)$$

$$p(t) = \cos(w_0 t)$$

$$P(jw) = \pi\delta(w - w_0) + \pi\delta(w + w_0)$$

$$R(jw) = \frac{1}{2\pi}[S(jw) * P(jw)]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\theta)P(j(w-\theta))d\theta$$

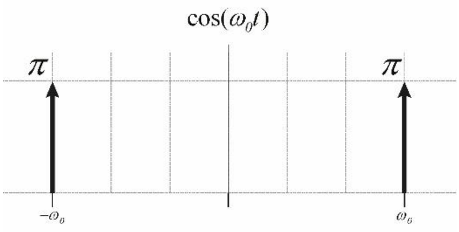
$$= \frac{1}{2}S\left(j(w - w_0)\right) + \frac{1}{2}S\left(j(w + w_0)\right)$$

Örnek $x(t) = \cos(\omega_0 t) * s(t)$ ***(Bkz.**Chapter4.pdf / Example 4.2)*

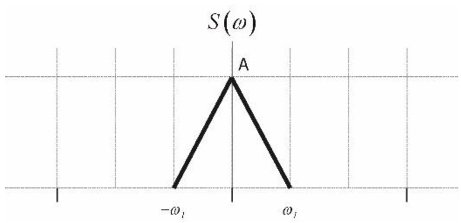
Çözüm $x(t) = \underbrace{\cos(\omega_0 t)}_{x_1(t)} * s(t)$

$$X(\omega) = \frac{1}{2\pi}\left[X_1(\omega) * S(\omega)\right]$$

$\frac{1}{2\pi}\Big[$

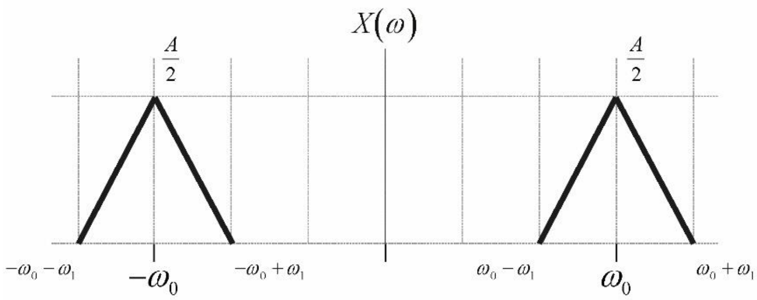


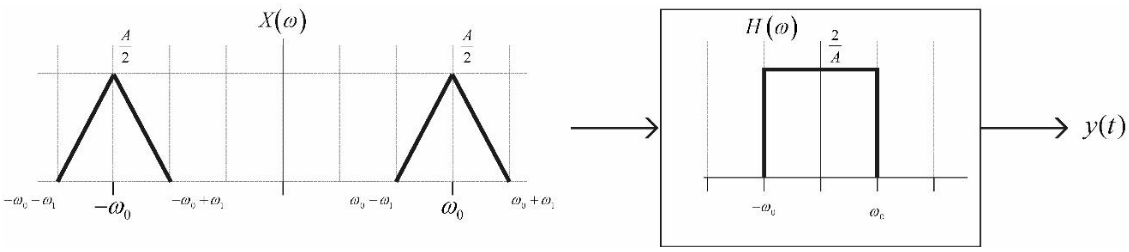
$*$



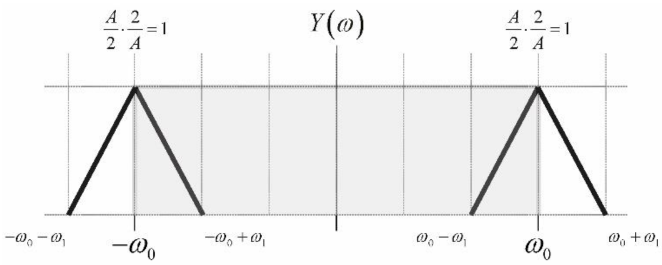
$\Big]$

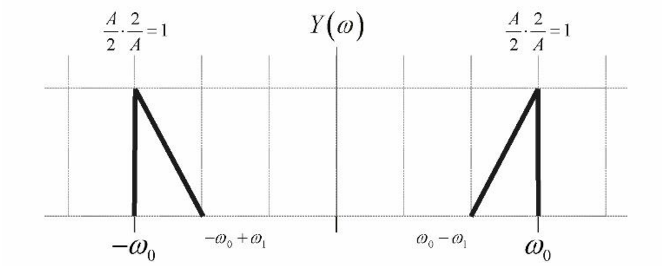
$X(\omega) =$





Örnek





***Örnek** Chapter4.pdf / Example 4.23*

- **Example 4.23:**

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(jw)e^{jwt}dw$$

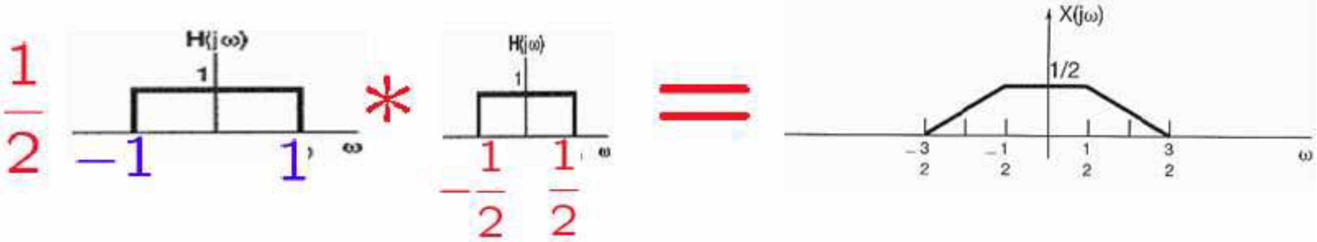
$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$x(t) = \frac{\sin(t)\sin(t/2)}{\pi t^2}$$

$$X(jw) = \int_{-\infty}^{\infty} \frac{\sin(t)\sin(t/2)}{\pi t^2} e^{-jwt}dt$$

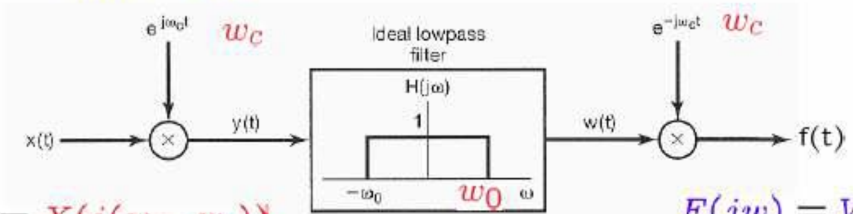
$$= \pi\left(\frac{\sin(t)}{\pi t}\right)\left(\frac{\sin(t/2)}{\pi t}\right)$$

$$\Rightarrow X(jw) = \frac{1}{2}\mathcal{F}\left\{\frac{\sin(t)}{\pi t}\right\} * \mathcal{F}\left\{\frac{\sin(t/2)}{\pi t}\right\}$$



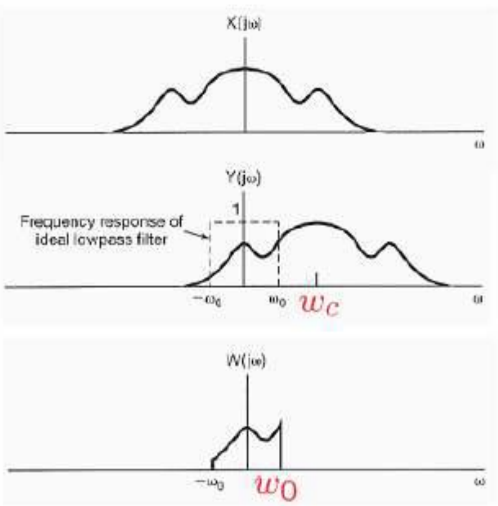
- **Bandpass Filter Using Amplitude Modulation:**

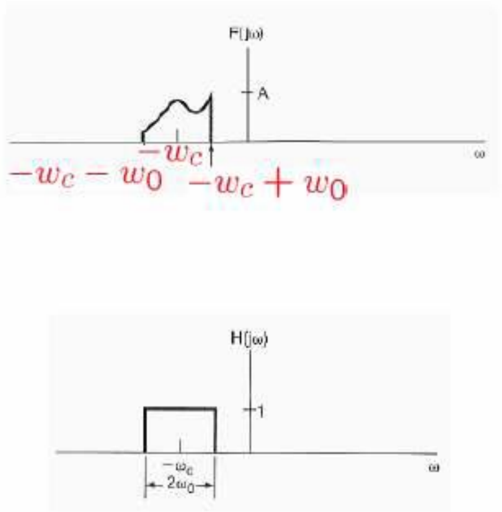
$$e^{jw_c t} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta(w - w_c)$$



$$Y(jw) = X(j(w - w_c))$$

$$F(jw) = W(j(w + w_c))$$



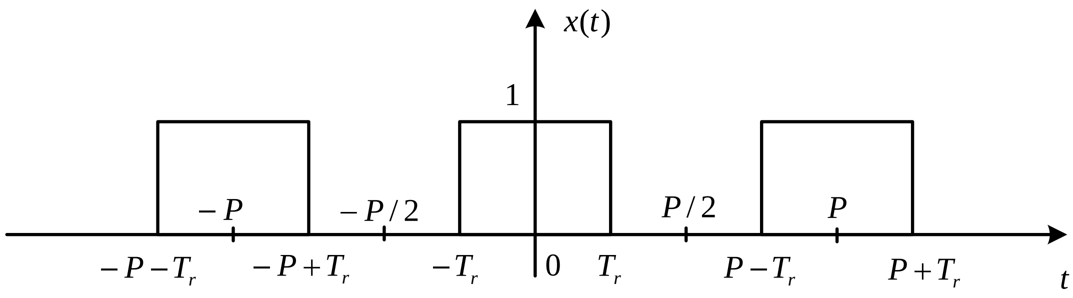


ÖRNEKLEME

(Bkz.IS_BOLUM3.docx)

Örnek 3.1

Şekil de gösterilen kare dalga işareti için Fourier serisi açılımını bulunuz.



İşaretin periyodu P olmaktadır. Böylece $\omega_0 = \frac{2\pi}{P}$ olarak bulunur. $x(t)$ çift-simetrik bir işaret olduğundan, integrali $(-\frac{P}{2}, \frac{P}{2})$ aralığında almak işlemi basitleştirecektir. İntegral sonucunu $k=0$ ve $k \neq 0$ için aşağıdaki şekilde elde ederiz.

$$a_k = \frac{1}{P}\int_{-P/2}^{P/2} x(t)\,.\,e^{-jk\omega_0 t}\,.\,dt \;=\; \frac{1}{P}\int_{-T_r}^{T_r} e^{-jk\omega_0 t}\,.\,dt \;=\; -\frac{1}{jk\omega_0 P}\cdot e^{-jk\omega_0 t}\Big|_{-T_r}^{T_r}, \quad k \neq 0 \text{ için,}$$

$$= \frac{2}{k\omega_0 P}\cdot\left(\frac{e^{jk\omega_0 T_r} - e^{-jk\omega_0 T_r}}{2j}\right) = \frac{2\sin(k\omega_0 T_r)}{k\omega_0 P}$$

$$a_0 = \frac{1}{P}\int_{-T_r}^{T_r} dt \;=\; \frac{2T_r}{P}, \quad k=0 \text{ için,}$$

Ayrıca a_0 terimini, L'Hopital kuralını kullanarak da aşağıdaki gibi bulabiliriz.

$$\lim_{k\to 0}\frac{2\sin(k\omega_0 T_r)}{k\omega_0 P} = \frac{2\omega_0 T_r \overbrace{\cos(k\omega_0 T_r)}^{1}}{\omega_0 P} = \frac{2T_r}{P}$$

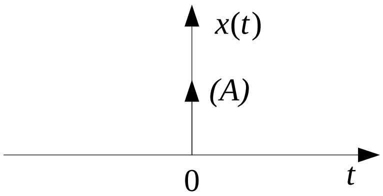
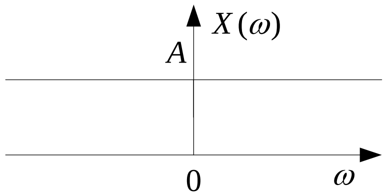
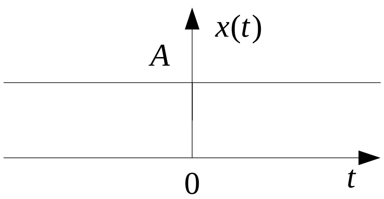
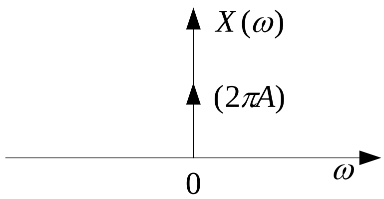
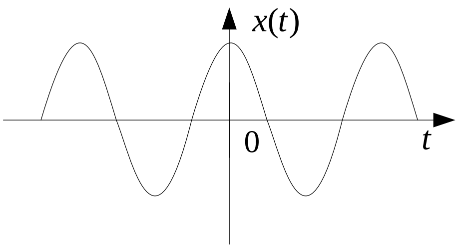
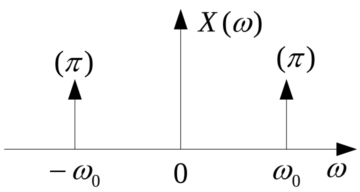
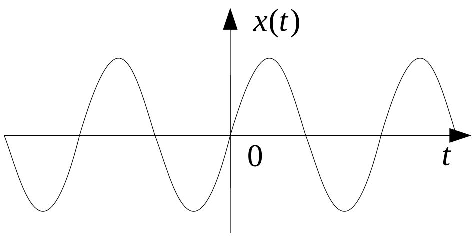
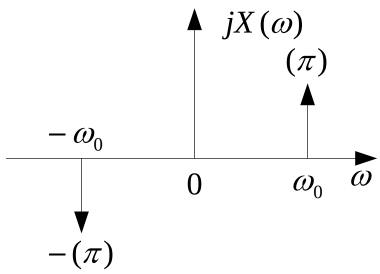
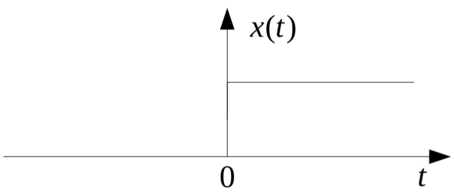
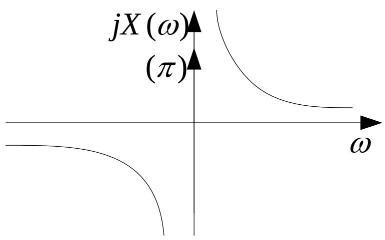
Böylece sonuç ifadesini genel olarak aşağıdaki gibi yazabiliriz.

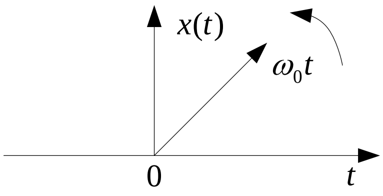
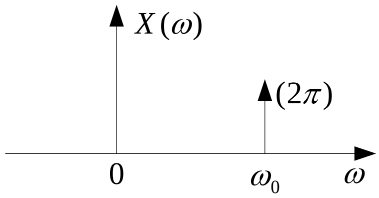
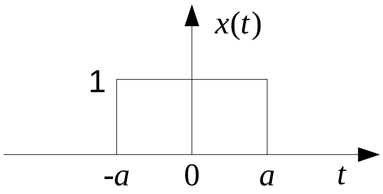
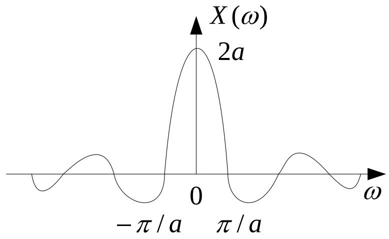
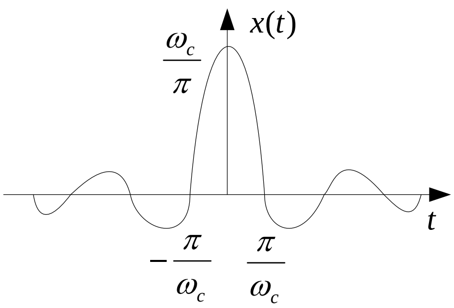
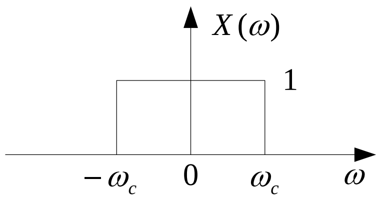
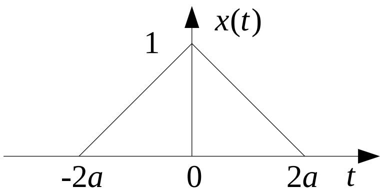
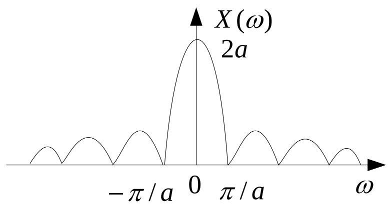
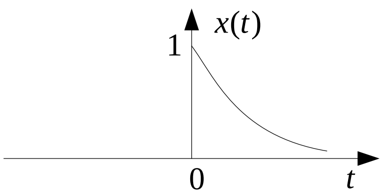
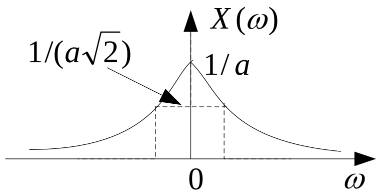
$$a_k = \frac{2\sin(k\omega_0 T_r)}{k\omega_0 P}$$

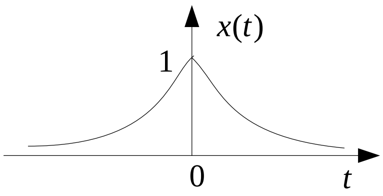
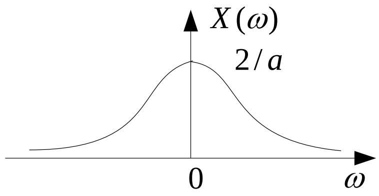
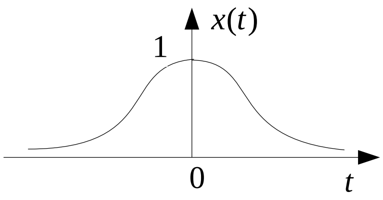
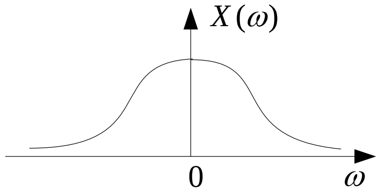
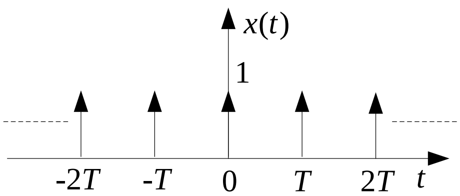
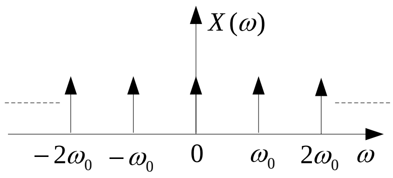
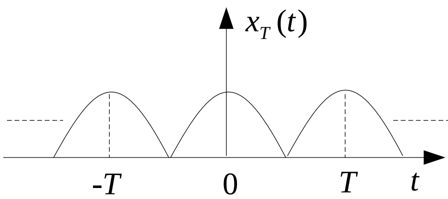
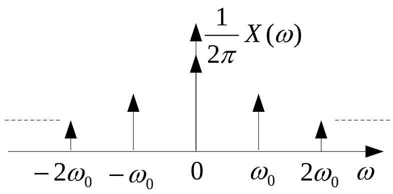
Bazı önemli Fourier dönüşümleri

1	Doğrusallık	$F[a.x(t)+b.y(t)]=a.X(\omega)+b.Y(\omega)$
2	Freakans kaydırma	$F[x(t).e^{j\omega_0 t}]=X(\omega-\omega_0)$
3	Zaman kaydırma	$F[x(t-t_0)]=X(\omega).e^{-j\omega t_0}$
4	Zaman türevi	$F\left[\frac{dx(t)}{dt}\right]=j\omega.X(\omega)$
5	Zaman integrali	$F\left[\int_{-\infty}^{t} x(\tau).d\tau\right]=\frac{X(\omega)}{j\omega}+\pi.X(0).\delta(\omega)$
6	Zaman domeninde konvolüsyon	$F[x(t)*y(t)]=X(\omega).Y(\omega)$ $x(t)*y(t)\triangleq\int_{-\infty}^{\infty} x(\tau).y(t-\tau).d\tau$
7	Frekans domeninde konvolüsyon	$F[X(\omega)*Y(\omega)]=x(t).y(t)$ $X(\omega)*Y(\omega)\triangleq\int_{-\infty}^{\infty} X(\alpha).Y(\omega-\alpha).d\alpha$
8	Ölçekleme	$F[x(at)]=\frac{1}{\lvert a\rvert}X(\frac{\omega}{a})$; a gerçel için
9	Perseval teoremi	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2.dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert x(\omega)\rvert^2.d\omega$
10	Dualite(zaman-frekans)	$F[x(t)]=2\pi X(-\omega)$
11	Korelasyon	$F\int_{-\infty}^{\infty} x(t).y(t+\tau).dt=F[x(t)*y(-t)]=X(\omega).Y^*(\omega)$
12	Karmaşık eşlenik	$F[x^*(t)]=X^*(-\omega)$
13	Genlik modülasyonu	$F[x(t).\cos\omega_0 t]=\frac{1}{2}X(\omega+\omega_0)+\frac{1}{2}X(\omega-\omega_0)$
14	Simetrik(çift-tek)	$F[x_{çift}(t)]=X_{çift}(\omega)$ $F[x_{tek}(t)]=X_{tek}(\omega)$
15	Frekans türevi	$F[t.x(t)]=j\frac{dX(\omega)}{d\omega}$
16	Gerçel $x(t)$	$X(\omega)=X^*(-\omega)$ $\mathrm{Re}[X(\omega)]=\mathrm{Re}[X(-\omega)]$ $\mathrm{Im}[X(\omega)]=-\mathrm{Im}[X(-\omega)]$ $\lvert X(-\omega)\rvert=-\lvert X(\omega)\rvert$ $\angle X(-\omega)=-\angle X(\omega)$

Bazı önemli Fourier dönüşümleri

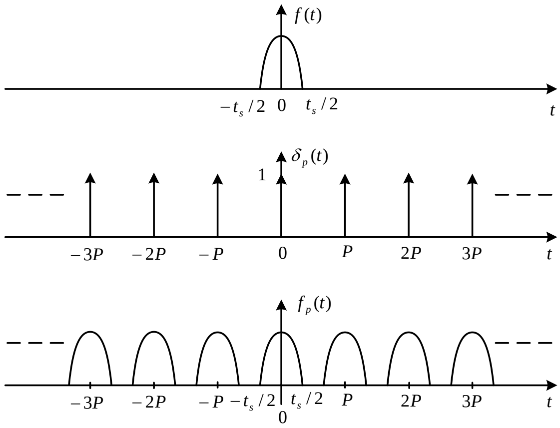
1	İmpuls	 $x(t) = A\delta(t)$	 $X(\omega) = A$
2	Sabit	 $x(t) = A$	 $X(\omega) = 2\pi A\delta(\omega)$
3	Kosinüs	 $x(t) = \cos(\omega_0 t)$	 $X(\omega) = \pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$
4	Sinüs	 $x(t) = \sin(\omega_0 t)$	 $X(\omega) = \pi(\delta(\omega-\omega_0)-\delta(\omega+\omega_0))$
5	Basamak	 $x(t) = u(t) = \int_{-\infty}^{t}\delta(\tau).d\tau$	 $X(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$

6	Karmaşık üstel	 $x(t) = e^{j\omega_0 t}$	 $X(\omega) = 2\pi\delta(\omega - \omega_0)$
7	Darbe	 $x(t) = u(t+a) - u(t-a)$	 $X(\omega) = 2a\dfrac{\sin(\omega a)}{\omega a}$
8	Sınırlı bantlı işaret	 $x(t) = \dfrac{\omega_c}{\pi}\dfrac{\sin(\omega_c t)}{\omega_c t}$	 $X(\omega) = u(\omega + \omega_c) - u(\omega - \omega_c)$
9	Üçgen	 $x(t) = 1 - \dfrac{1}{2a}\lvert t \rvert;\ \ \lvert t \rvert < 2a$	 $X(\omega) = 2a\dfrac{\sin^2(\omega a)}{(\omega a)^2}$
10	Tek taraflı üstel işaret	 $x(t) = e^{-at}.u(t);\ \ a > 0$	 $X(\omega) = \dfrac{1}{a + j\omega}$

11	Çift taraflı üstel işaret	 $x(t)=e^{-a\|t\|}\ ;\ \ a>0$	 $X(\omega)=\dfrac{2a}{\omega^2+a^2}$
12	Gauss işareti	 $x(t)=e^{-at^2}$	 $X(\omega)=\sqrt{\dfrac{\pi}{a}}.e^{\omega^2 4a}$
13	İmpuls treni	 $x(t)=x_T(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$	 $X(\omega)=\dfrac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$ $\omega_0\equiv\dfrac{2\pi}{T}$
14	Periyodik işaret	 $x_T(t)=x_T(T+t)=\sum_{k=-\infty}^{\infty}X_k.e^{jk\omega_0 t}$ X_k :Fourier Serisi Katsayıları	 $X(\omega)=2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$ $\omega_0\equiv\dfrac{2\pi}{T}$

ÖRNEKLEME

$$x_s(t) = x_a(t) * s(t)$$



$x_a(t)$ Analog ifade

T_s Örnekleme

$$x(n) = x_a(nT_s)$$

Örnek

$x_a(t) = e^{j\omega_0 t}$ Analog ifade T_s Örnekleme $x(n) = ?$

Çözüm

$$x(n) = x_a(nT_s) = e^{jw_0(nT_s)}$$

Periyodik midir?

$$x(n) = x(n+N)$$
$$e^{jw_0 nT_s} = e^{jw_0(n+N)T_s}$$
$$\cancel{e^{jw_0 nT_s}} = \cancel{e^{jw_0 nT_s}} \cdot e^{jw_0 NT_s}$$
$$1 = e^{jw_0 NT_s}$$
$$e^{j2\pi k} = e^{jw_0 NT_s}$$

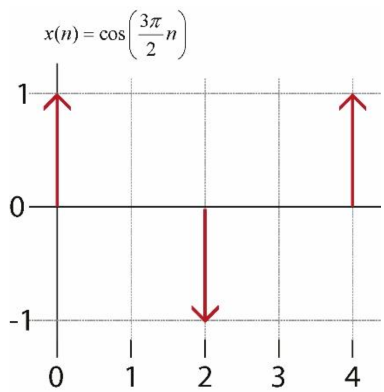
$$j2\pi k = jw_0 NT_s$$
$$N = \frac{\cancel{j}2\pi k}{\cancel{j}w_0 T_s} = \frac{\cancel{2\pi} k}{\frac{\cancel{2\pi}}{T_0} T_s} = \frac{kT_0}{T_s}$$

Örnek

$x_a(t) = \cos(15t)$ Analog ifade

$T_s = \dfrac{\pi}{10} s$ Örnekleme

$x(n) = ?$ $N = ?$



Çözüm

$$x(n) = x_a(nT_s) = \cos(15nT_s) = \cos\left(15n\frac{\pi}{10}\right) = \cos\left(\frac{3\pi}{2} n\right)$$

$$N = k\frac{T_0}{T_s} = k\frac{\frac{2\pi}{\omega_0}}{\frac{\pi}{10}} = k\frac{2\pi}{15}\frac{10}{\pi} = \frac{4}{3}k$$

$k = 3$ ise Periyodu 4

Örnek $x_a(t)=e^{-\alpha t}$ Analog ifade T_s periyotlarla örnekleniyor $x(n)=?$ Elde edilen periyot?

Çözüm

$$x_a(t)=e^{-\alpha t}$$

$$x(n)=x_a(nT_s)=e^{-\alpha nT_s}$$

$$X(z)=\frac{1}{1-e^{-\alpha Ts}z^{-1}} \qquad |z|>e^{-\alpha T_s}$$

Özetle yapılan işlemler;

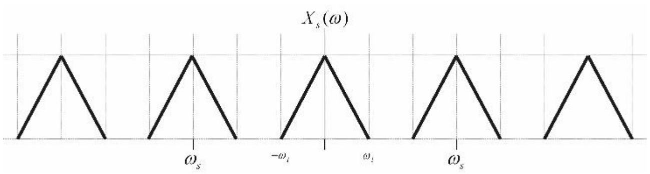
$$x_s(t)= x_a(t) \quad x \quad s(t)$$

Analog işaret impuls treni

Çarpma işlemi yapıyoruz frekans domeninde karşılığı konvolüsyon işlemi

$$s(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$

$$X_s(\omega)=\frac{1}{2\pi}\left[X_a(\omega)*S(\omega)\right]$$



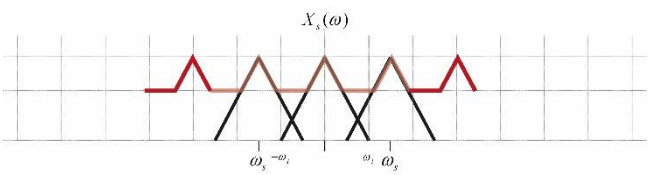
Bozulma olmasını istemiyorsak;

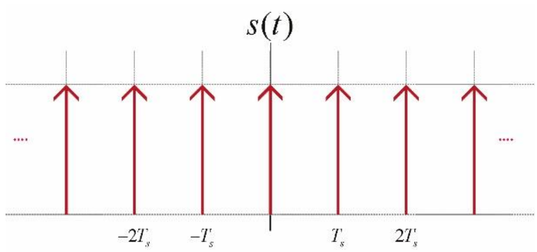
$$\omega_s-\omega_1>\omega_1$$

$$\omega_s>2\omega_1$$

Bu şartı sağlaması gerekiyor

Bu şart sağlanmazsa bozulmalar başlar



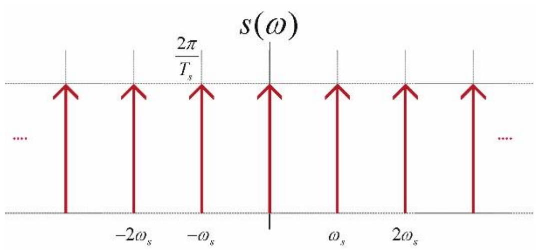


$$s(t)=\sum a_k e^{jk\omega_0 t}=\frac{1}{T_s}\sum_k e^{jk\omega_0 t}$$

$$s(\omega)=\frac{1}{T_s}\int_{-T_s/2}^{T_s/2}x_a(t)e^{-jk\omega_s t}dt=\frac{1}{T_s}\int\delta(t)e^{-jk\omega_s t}dt=\frac{1}{T_s}$$

$$\sum a_k 2\pi\delta(\omega-k\omega_0)$$

$$s(\omega)=\frac{2\pi}{T_s}\sum\delta(\omega-k\omega_0)$$



Örnek $x(n)=\cos\left(n\dfrac{\pi}{8}\right) \qquad f_s=10kHz \qquad x_a(t)=?$

Çözüm

$$x(n)=x_a(nT_s)$$

$$\cos\left(n\frac{\pi}{8}\right)=\cos\left(\omega_0 nT_s\right)$$

$$n\frac{\pi}{8}=\omega_0 nT_s$$

$$\frac{\pi}{8}=\omega_0 T_s$$

$$\frac{\pi}{8}=\omega_0\frac{1}{f_s}$$

$$\omega_0=\frac{\pi}{8}f_s=\frac{\pi}{8}\left(10.10^3\right)=1250\pi$$

$$x_a(t)=\cos(1250\pi t)$$

Örnek $\omega_s>2\omega_1$ *bu örnekte k=-1 değeri için analog işaret bulunmuş*

Amaç f_0 ‘ı bulmak

$$\cos\left(\omega_0 nT_s\right)=\cos\left(n\frac{\pi}{8}\right)$$

$$\cos\Bigg(\underbrace{\underbrace{2\pi f_0}_{\omega_0} n\frac{1}{f_s}}_{\frac{\pi}{8}}\Bigg)=\cos\left(n\frac{\pi}{8}\right)$$

$$\cos\left(\frac{2\pi f_0}{f_s}n+2\pi kn\underbrace{\frac{f_s}{f_s}}_{1}\right)=\cos\left(n\frac{\pi}{8}\right)$$

$$\frac{2\pi f_0}{f_s}n+2\pi kn\underbrace{\frac{f_s}{f_s}}_{1}=n\frac{\pi}{8}$$

$$\frac{2\not{n}\not{\pi}\left(f_0+kf_s\right)}{f_s}=\frac{\not{n}\not{\pi}}{8}$$

$$\frac{f_0+kf_s}{f_s}=\frac{1}{16}$$

$$f_0=\frac{f_s}{16}-\underbrace{k}_{-1\text{ için}}f_s$$

$$=\frac{f_s}{16}+f_s$$

$$=\frac{10000}{16}+10000$$

$$=625+10000$$

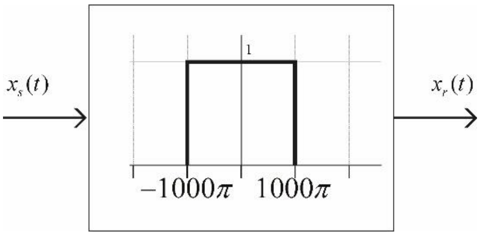
$$=10625\ Hz$$

$$\cos\left(2\pi f_0 t\right)$$

$$\cos\left(2\pi\cdot 10625t\right)$$

$$\cos\left(21250\pi t\right)$$

Örnek $x_a(t)=\cos\left(500\pi t+\frac{\pi}{4}\right)$ $\quad \omega_s = 2000\pi$ *örnekleme frekansı ile örnekleniyor* $x_r(t)=?$



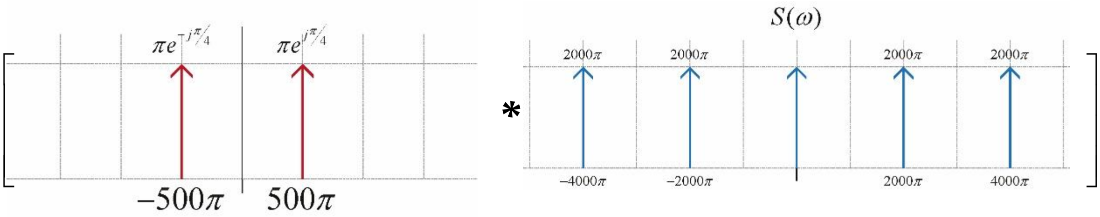
Çözüm

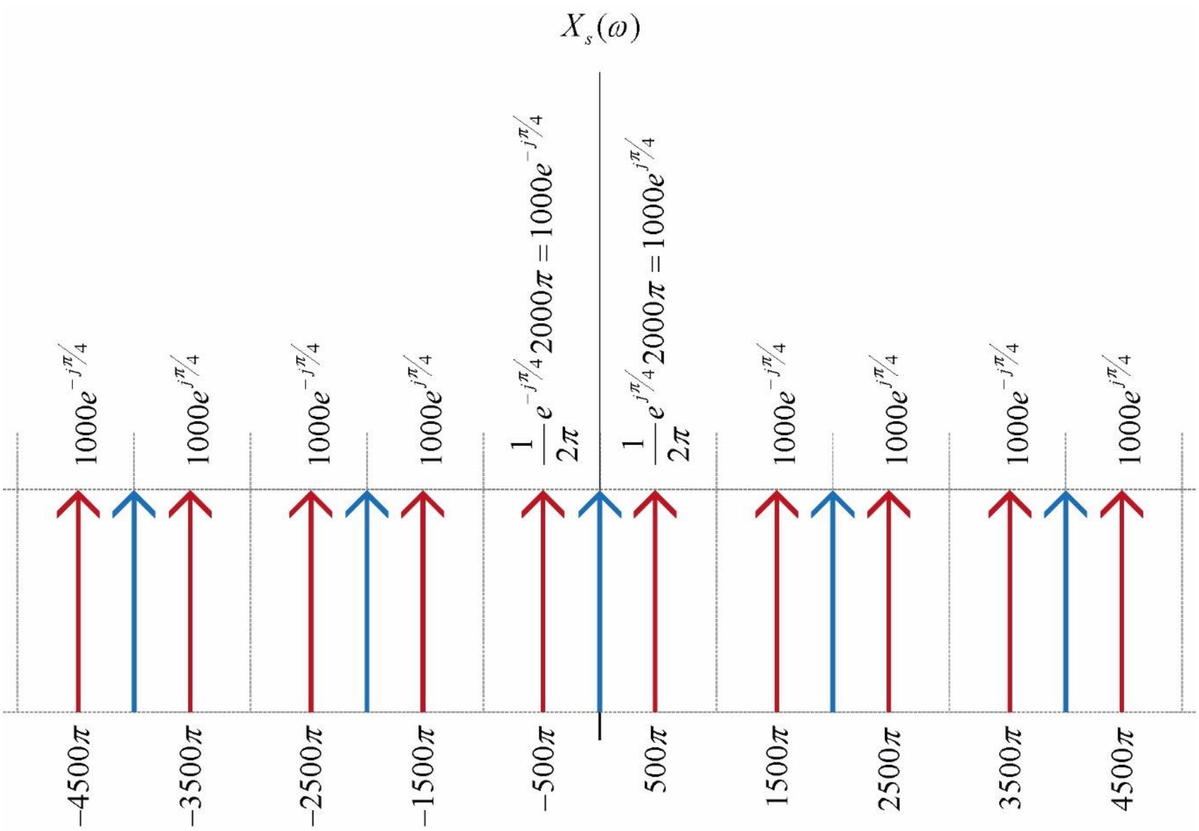
$$x_a(t)=\cos\left(500\pi t+\frac{\pi}{4}\right) \qquad x_s(t)=x_a(t)\ x\ s(t)$$

$$a_1=\frac{e^{j\pi/4}}{2} \qquad a_{-1}=\frac{e^{-j\pi/4}}{2} \qquad \text{Periyot}=\frac{\pi}{4} \qquad \omega_0=500\pi$$

$$X_s(\omega)=\frac{1}{2\pi}\left[X_a(\omega)*S(\omega)\right]$$

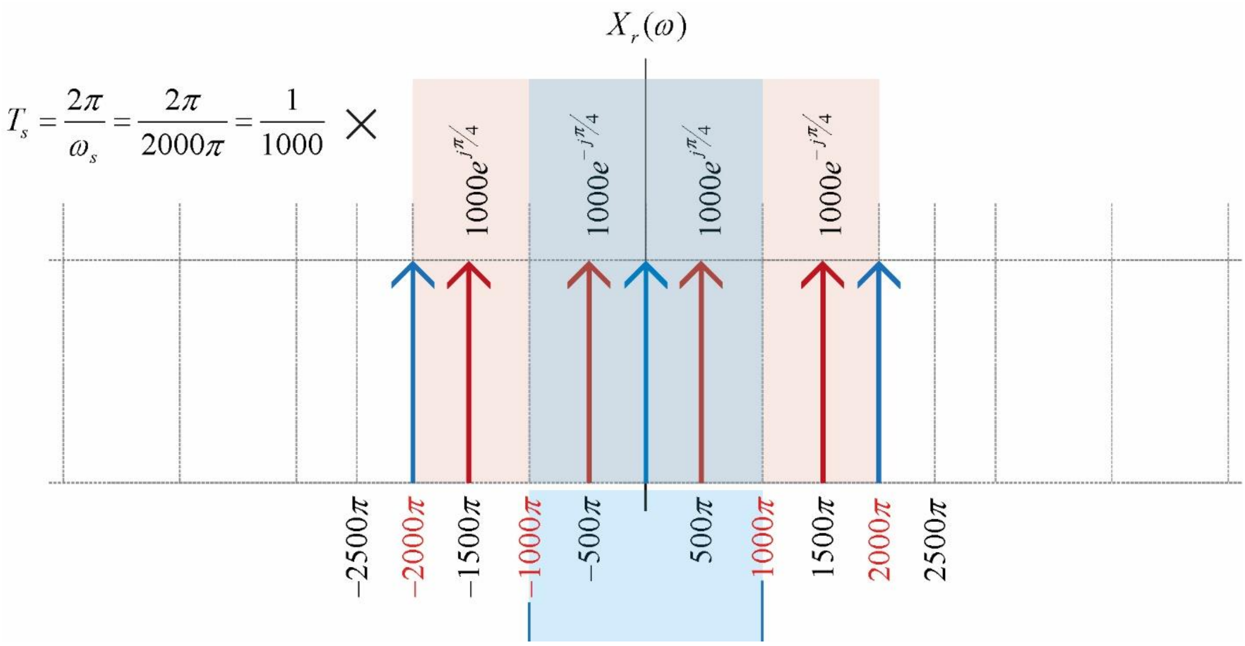
$X_s(\omega)=\frac{1}{2\pi}$

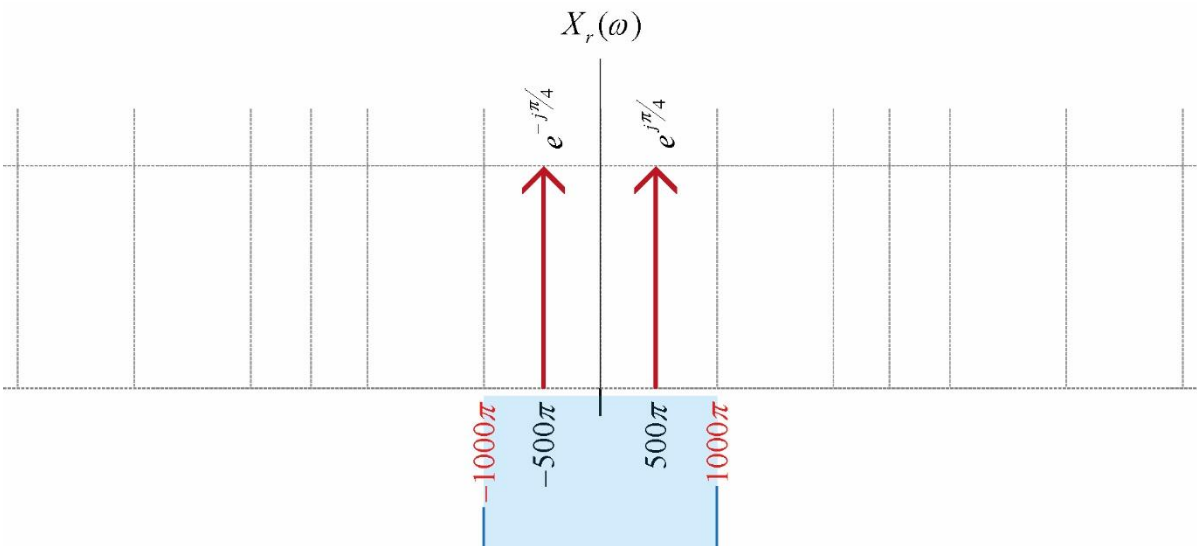




$$T_s=\frac{2\pi}{\omega_s}=\frac{2\pi}{2000\pi}=\frac{1}{1000}$$

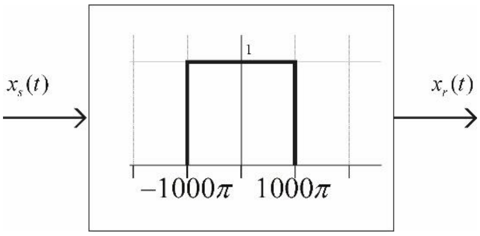
$$T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{2000\pi} = \frac{1}{1000} \times$$





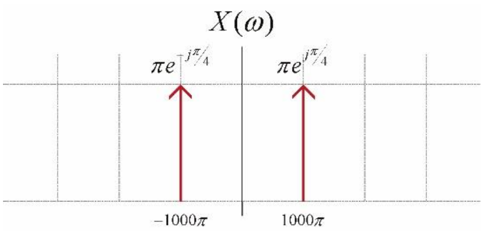
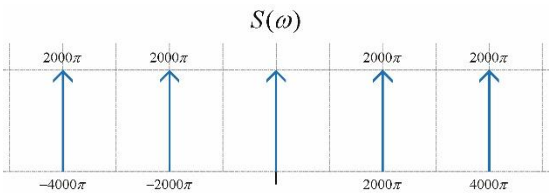
$$X_r(t) = \cos\left(500\pi + \frac{\pi}{4}\right)$$

Örnek $x_a(t) = \cos\left(1000\pi t + \frac{\pi}{4}\right)$



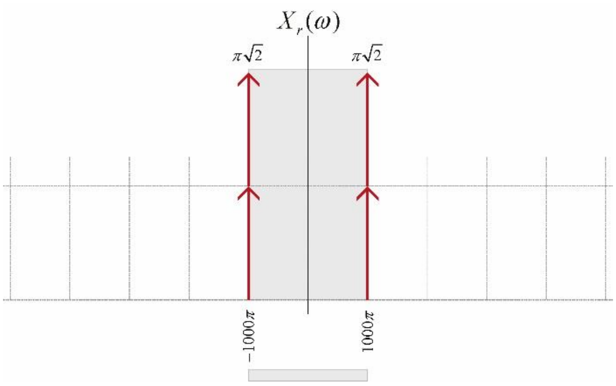
$$x_s(t) = x_a(t)\ x\ s(t)$$

$$X_s(\omega) = \frac{1}{2\pi}\left[X_a(\omega) * S(\omega)\right]$$

$X_s(\omega) = \frac{1}{2\pi}\Big[$  $*$  $\Big]$

$$2000\pi\left(\frac{e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}}{2}\right) = 2000\pi\cos\left(\frac{\pi}{4}\right) = 2000\pi\frac{\sqrt{2}}{2} = 1000\pi\sqrt{2}$$

$$a_1 = \frac{e^{j\frac{\pi}{4}}}{2} \qquad a_{-1} = \frac{e^{-j\frac{\pi}{4}}}{2}$$



$$x_r(t) = \sqrt{2}\cos(1000\pi t)$$

?????— ________________

$$T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{1000\pi\sqrt{2}} = \frac{\sqrt{2}}{1000}$$

$$\frac{1}{2\pi}\pi e^{-j\frac{\pi}{4}} 2000\pi = 1000\pi e^{-j\frac{\pi}{4}} x2$$

21 Temmuz 2017 (son ders) Özet &Geçmiş Final (2016) Sorularının çözümü

Kısmi Kesirleme

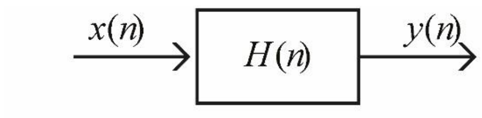
Payın derecesi ≥ Paydanın derecesi

$$\frac{N(z)}{\pi\left(1-P_i z^{-1}\right)} \Rightarrow \frac{A}{1-P_1 z^{-1}}+\frac{B}{1-P_2 z^{-1}}+\cdots+\frac{Y}{1-P_i z^{-1}}$$

$$\alpha u(n) \qquad \frac{1}{1-\alpha z^{-1}} \qquad |z|>|\alpha|$$

$$-\alpha u(-n-1) \qquad \frac{1}{1-\alpha z^{-1}} \qquad |z|<|\alpha|$$

Ayrık Zamanlı Sistem



$$y(n)=x(n)*h(n)$$
$$Y(z)=X(z)\cdot H(z)$$
$$H(z)=\frac{Y(z)}{X(z)}$$

$$y(n)-y(n-1)=x(n)+x(n-1)$$
$$Y(z)-z^{-1}Y(z)=X(z)+z^{-1}X(z)$$
$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+z^{-1}}{1-z^{-1}}$$

$$\begin{array}{r|l} z^{-1}+1 & z^{-1}+1 \\ -\;\; z^{-1}+1 & -1 \\ \hline 2 & \end{array}$$

Bölme işlemi yapılır

yada özellikler kullanılır

$$H(z)=-1+\frac{2}{1-z^{-1}}$$
$$h(n)=-\delta(n)+2u(n)$$

$$H(z)=\frac{1+z^{-1}}{1-z^{-1}}$$
$$H(z)=\frac{1}{1-z^{-1}}+\frac{z^{-1}}{1-z^{-1}}$$
$$h(n)=u(n)+u(n-1)$$

Fourier Serisi

Periyodik ve sürekli

$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \cos(\omega_0 t)=\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2} \qquad \sin(\omega_0 t)=\frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}$$

Periyodik ise;	*Periyodik değil ise;*
$a_k=\frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$ $X(\omega)=\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega-k\omega_0)$	*Fourier dönüşümü yapılır* $X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}$ $x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$

***Örnekleme** (Çarpma İşlemidir)*

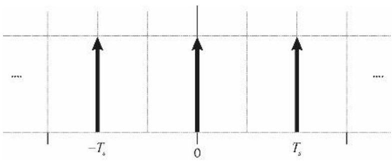
$$r(t)=s(t)\ x\ p(t)$$

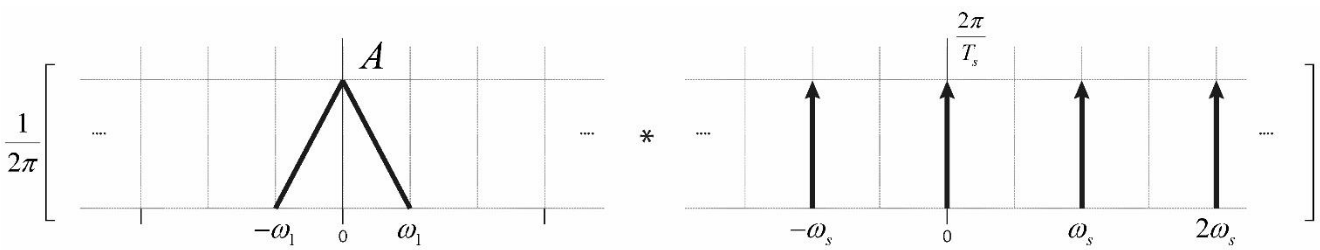
$$R(\omega)=\frac{1}{2\pi}\left[S(\omega)*P(\omega)\right]$$

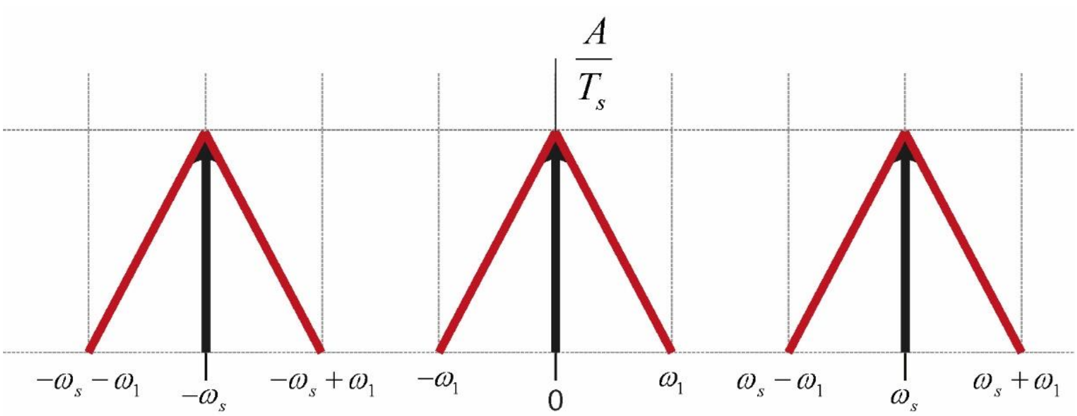
$$x_s(t)=x_a(t)\ x\ s(t)$$

$$s(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT_s)=\frac{1}{T_s}\sum_k e^{jk\omega_s t}$$

$$S(\omega)=\frac{2\pi}{T_s}\sum_k \delta(\omega-k\omega_s)$$







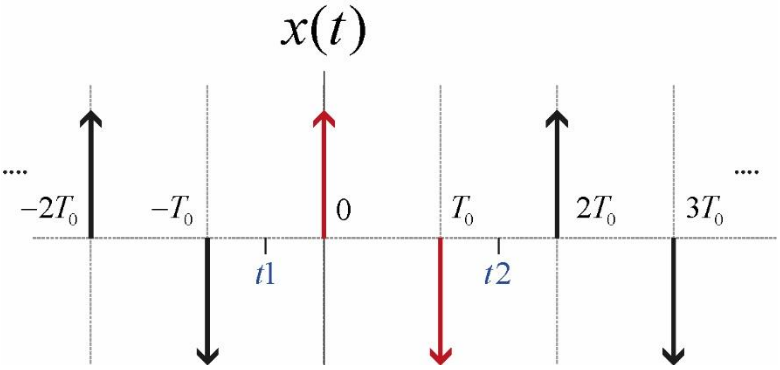
$$\left.\begin{aligned}\omega_s-\omega_1 &> \omega_1\\ \omega_s &> 2\omega_1\end{aligned}\right\}$$

*Örtüşme **olmama**sı için bu şart sağlanmalı,*
Sağlanmaz ise örtüşmeler başlıyor orijinal işaret elde edilemez hale geliyor.

$x_a(t)$ *Analog zaman işareti*

T_s *Örnekleme*

$x(n) = x_a(nT_s)$



$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_s t}dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{3T_0}{2}} \left(\delta(t) - \delta(t - T_0)\right)e^{-jk\omega_0 t}dt$$

$$= \frac{1}{2T_0}\left[\int \delta(t)e^{-jk\omega_0 t}dt - \int \delta(t - T_0)e^{-jk\omega_0 t}dt\right]$$

$$= \frac{1}{2T_0}\left[1 - e^{-jk\frac{\pi}{T_0}T_0}\right]$$

$$= \frac{1}{2T_0}\left(1 - e^{-jk\pi}\right)$$

$$= \frac{1}{2T_0}\left(1 - (-1)^k\right)$$

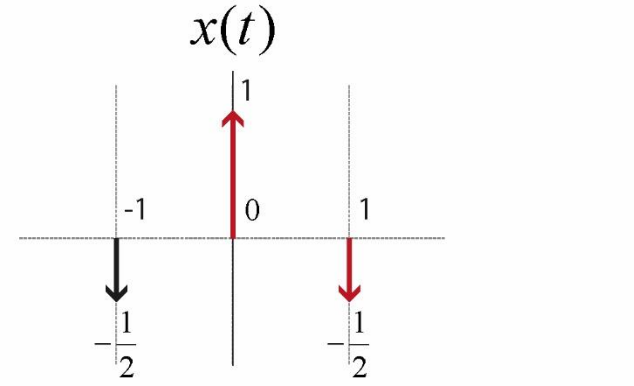
$$\omega_0 = \frac{2\pi}{Periyot} = \frac{2\pi}{2T_0} = \frac{\pi}{T_0}$$

$$t1 = \frac{3T_0}{2}$$

$$t2 = -\frac{T_0}{2}$$

$$\frac{1}{2T_0}\left(1 - (-1)^k\right) = \begin{cases} \dfrac{1}{T_0} & k\ tek \\ 0 & k\ çift \end{cases}$$

Örnek



$X(\omega)=?$

Çözüm

$T=2$

$\omega_0=\frac{2\pi}{T}=\frac{2\pi}{2}=\pi$

$$X(\omega)=\int_{-\infty}^{\infty}\left(-\frac{1}{2}\underset{t=-1}{\delta(t+1)}+\underset{t=0}{\delta(t)}-\frac{1}{2}\underset{t=1}{\delta(t-1)}\right)e^{-j\omega t}d\omega$$

$$=-\frac{1}{2}e^{j\omega}+1-\frac{1}{2}e^{-j\omega}$$

$$=1-\frac{e^{j\omega}+e^{-j\omega}}{2}$$

$$=1-\cos(\omega)$$

Örnek $x(t)=\sum_{k=-\infty}^{\infty}x(t-kT_1)$ $X(\omega)=?$

Çözüm

$Periyot=T$

$\omega_0=\frac{2\pi}{T_1}$

$$a_k=\frac{1}{T_1}\int\left(-\frac{1}{2}\underset{t=-1}{\delta(t+1)}+\underset{t=0}{\delta(t)}-\frac{1}{2}\underset{t=1}{\delta(t-1)}\right)e^{-jk\omega_0 t}dt$$

$$=\frac{1}{T_1}\left(-\frac{1}{2}e^{jk\omega_0}+1-\frac{1}{2}e^{-jk\omega_0}\right)$$

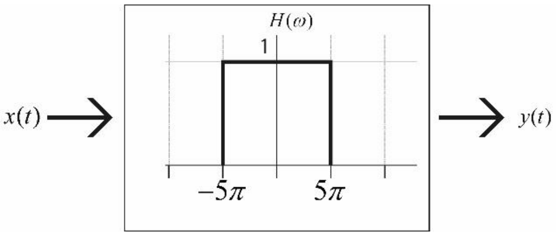
$$=\frac{1}{T_1}\left(1-\cos(k\omega_0)\right)$$

2016 BSM307 Final 1.Soru Temel frekansı $\omega_0 = 2\pi$ olarak verilen $x(t)$ işaretinin fourier seri katsayıları

$a_0 = 1$ $\quad a_1 = a_{-1} = \frac{1}{4}$ $\quad a_2 = a_{-2} = \frac{1}{2}$ $\quad a_3 = a_{-3} = \frac{1}{3}$ 'tür

$x(t)$ işaretini aşağıda spektrumları verilen sistemlere uyguladığımızda çıkışında elde edeceğimiz $y(t)$ işaretinin temel frekansını ve fourier seri katsayılarını yazınız.

a)



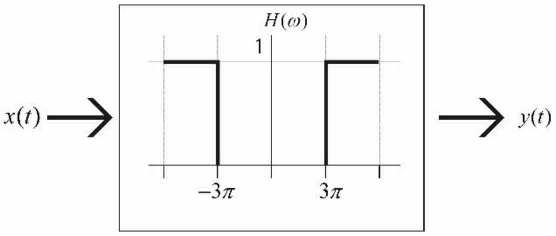
Temel Frekans $\omega_0 = 2\pi$

$a_0 = 1$

$a_1 = a_{-1} = \frac{1}{4}$

$a_2 = a_{-2} = \frac{1}{2}$

b)

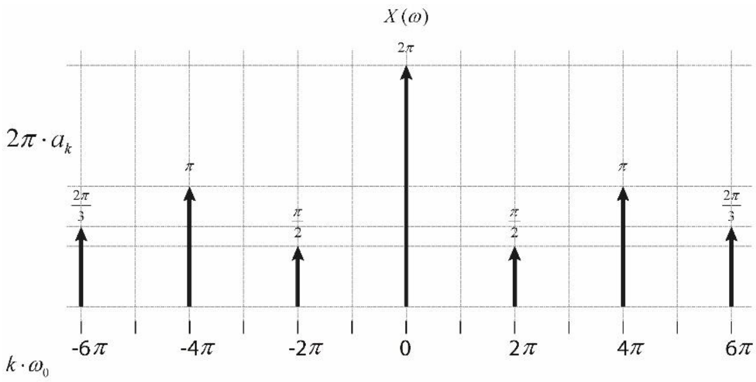


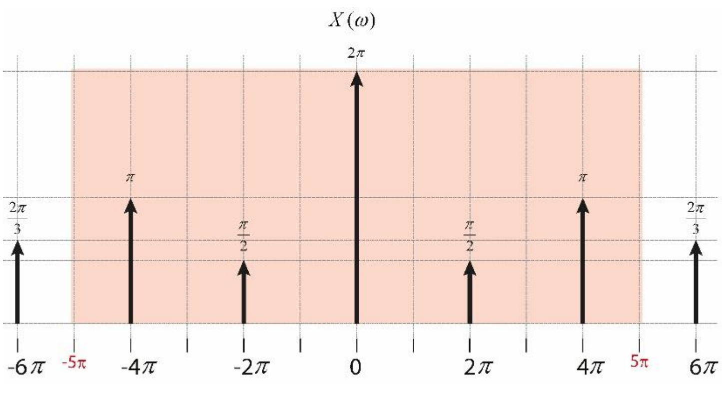
Temel Frekans $\omega_0 = 2\pi$

$a_2 = a_{-2} = \frac{1}{3}$

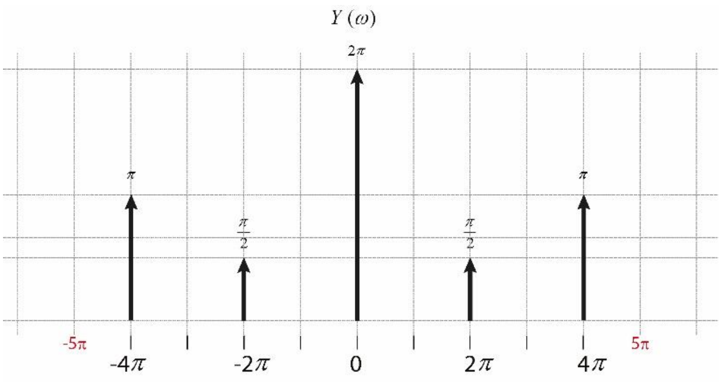
$a_3 = a_{-3} = \frac{1}{3}$

2016 BSM307 Final 1.Soru Çözüm





a.)

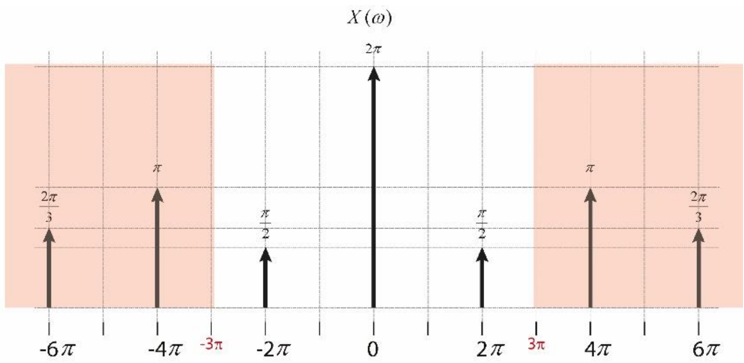


$Temel\ Frekans\ \omega_0 = 2\pi$

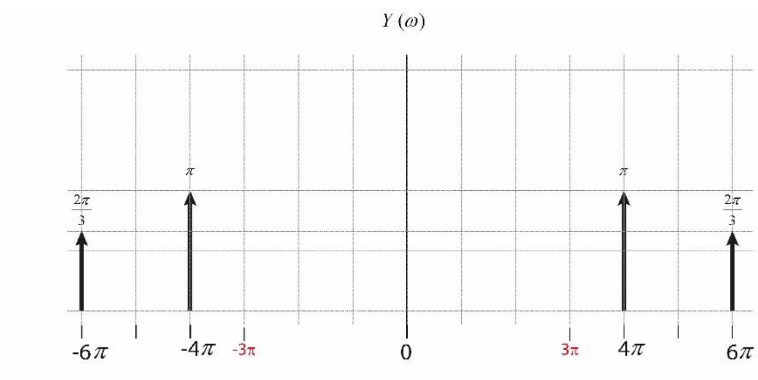
$a_0 = 1$

$a_1 = a_{-1} = \frac{1}{4}$

$a_2 = a_{-2} = \frac{1}{2}$



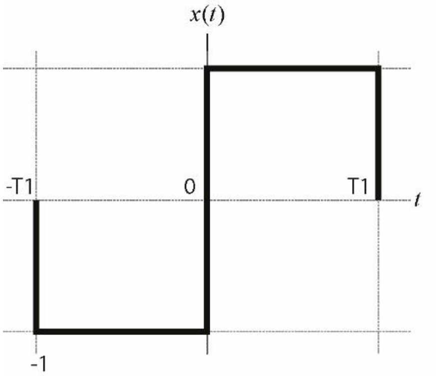
b.)



$Temel\ Frekans\ \omega_0 = 2\pi$

$a_2 = a_{-2} = \frac{1}{3}$

$a_3 = a_{-3} = \frac{1}{3}$



2016 BSM307 Final 2.Soru

$$X(\omega)=?$$

2016 BSM307 Final 2.Soru Çözüm

$$X(\omega)=\int_{-T_1}^{0}-1e^{-j\omega t}dt+\int_{0}^{T_1}1e^{-j\omega t}dt$$

$$=\frac{1}{j\omega}\cdot e^{-j\omega t}\Big|_{-T_1}^{0}+\frac{1}{-j\omega}e^{-j\omega t}\Big|_{0}^{T_1}$$

$$=\frac{1}{j\omega}\left(1-e^{j\omega T_1}\right)+\frac{1}{j\omega}\left(-e^{-j\omega T_1}+1\right)$$

$$=\frac{1}{j\omega}-\frac{1}{j\omega}e^{j\omega T_1}-\frac{1}{j\omega}e^{-j\omega T_1}+\frac{1}{j\omega}$$

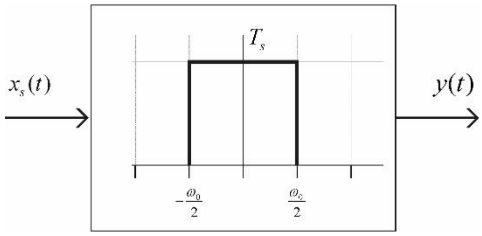
$$=\frac{2}{j\omega}-\frac{1}{j\omega}\left(e^{j\omega T_1}+e^{-j\omega T_1}\right)$$

$$=\frac{2}{j\omega}-\frac{2}{j\omega}\left(\frac{e^{j\omega T_1}+e^{-j\omega T_1}}{2}\right)$$

$$=\frac{2}{j\omega}\left(1-\cos\left(\omega T_1\right)\right)$$

2011 BSM307 Örnek Final 3.Soru $x(t)=\cos\left(1000\pi t+\frac{\pi}{2}\right) \qquad T_s=1ms$

2011 BSM307 Örnek Final 3.Soru Çözüm

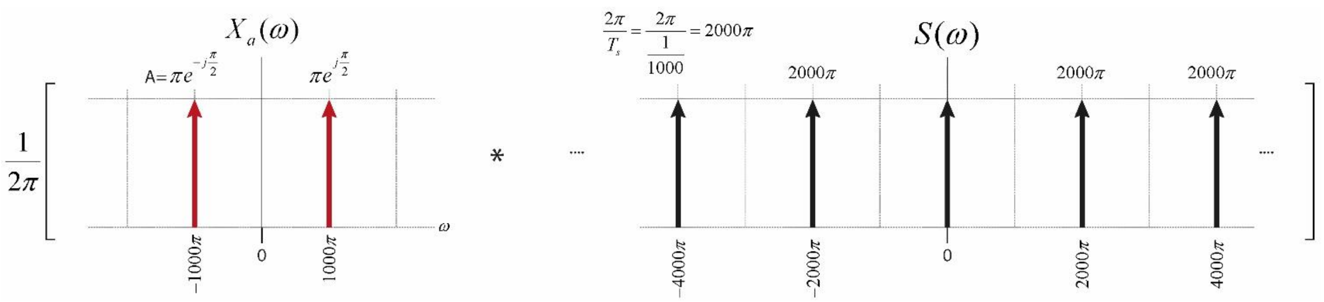


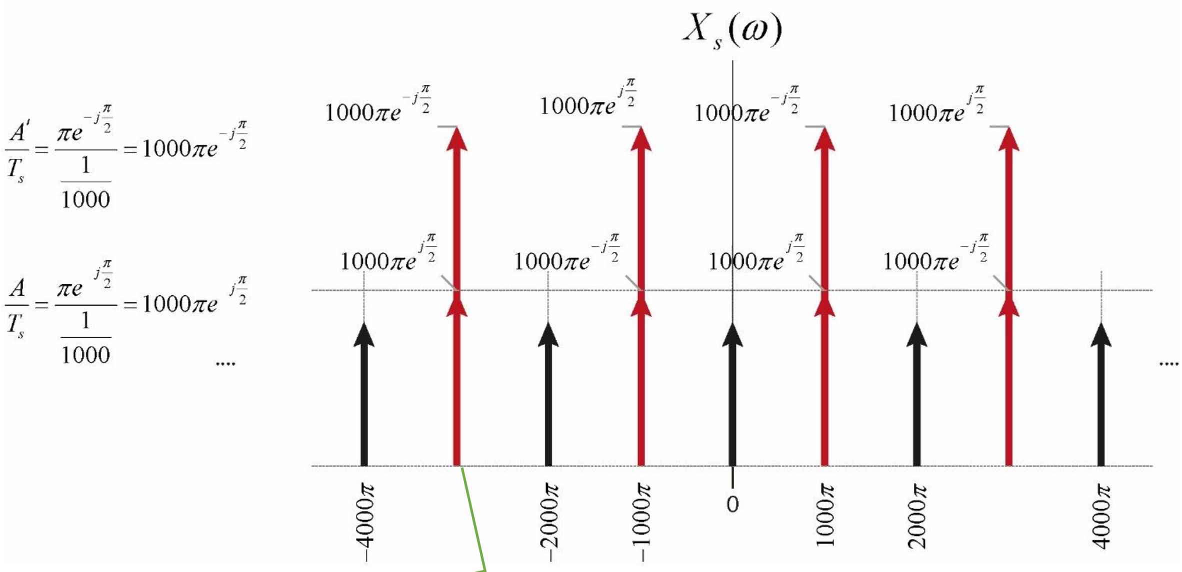
$$x_s(t)=x_a(t)\ x\ s(t)$$

$$X_s(\omega)=\frac{1}{2\pi}\left[X_a(\omega)*S(\omega)\right]$$

$$T_s=1ms=\frac{1}{1000}sn=1000Hz$$

$$\omega_s=\frac{2\pi}{T_s}=\frac{2\pi}{\frac{1}{1000}}=2000\pi \qquad \omega_0=1000\pi \qquad Periyot=T_0=\frac{\pi}{2}$$





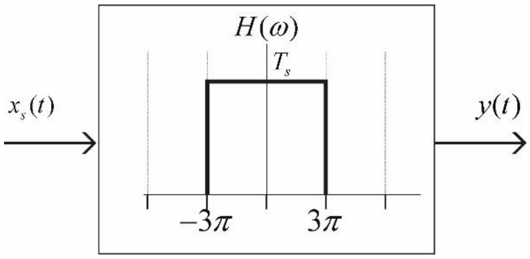
$$\begin{aligned}
A+A' &= 1000\pi e^{j\frac{\pi}{2}}+1000\pi e^{-j\frac{\pi}{2}} \\
&= 1000\pi\left(e^{j\frac{\pi}{2}}+e^{-j\frac{\pi}{2}}\right) \\
&= 2\cdot 1000\pi\left(\frac{e^{j\frac{\pi}{2}}+e^{-j\frac{\pi}{2}}}{2}\right) \\
&= 2000\pi\cos\left(\frac{\pi}{2}\right) \\
&= 2000\pi\underbrace{\cos(90)}_{0} \\
&= 0
\end{aligned}$$

$$X_s(\omega)=0$$

$$Y(\omega)=0\rightarrow y(t)=0$$

Örnek 4.Soru $x_a(t) = e^{-j\pi t} - e^{-j2\pi t}$ $\quad T_s = \frac{2}{3}s$

$y(t) = ?$

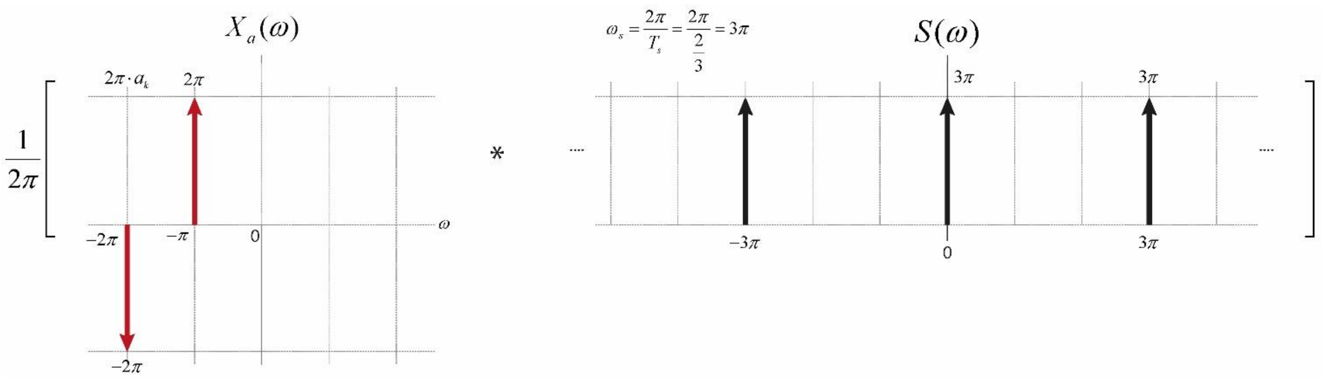


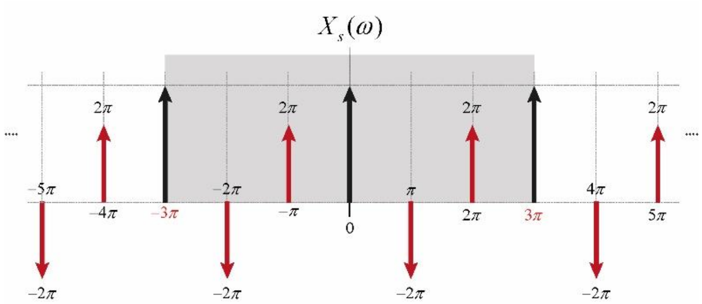
Örnek 4.Soru. Çözüm

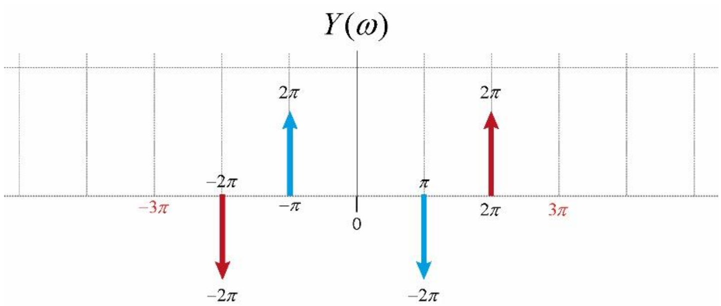
$$y(t) = x_s(t) \cdot h(t)$$
$$x_s(t) = x_a(t) \cdot s(t)$$
$$X_s(\omega) = \frac{1}{2\pi}\left[X_a(\omega) * S(\omega)\right]$$

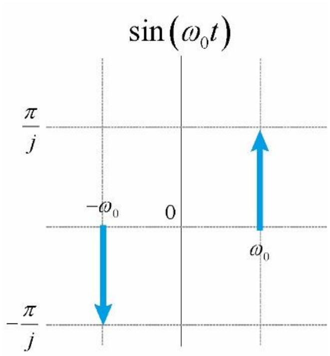
$$\omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{\frac{2}{3}} = 3\pi$$

$$\omega_0 = \pi$$
$$a_{-1} = 1$$
$$a_{-2} = -1$$





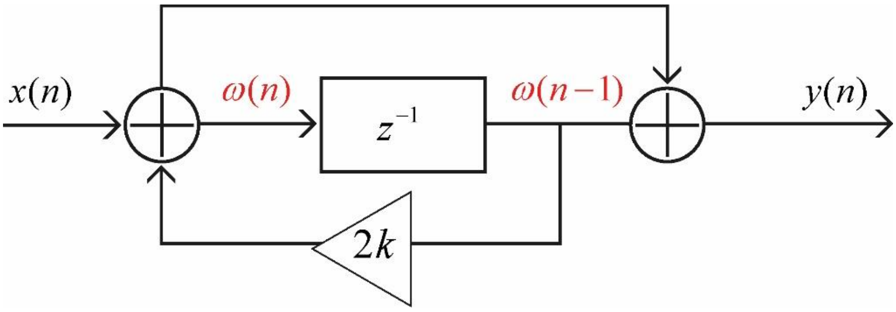




$$y(t) = -2j\sin(\pi t) + 2j\sin(2\pi t)$$

$$y(t) = \sin(\omega_0 t)$$

BSM307 Örnek Final 5.Soru



Hangi k değerlerinde sistem kararlıdır? Sistem Nedensel

BSM307 Örnek Final 5.Soru Çözüm

$$y(n)=\omega(n)+\omega(n-1)$$
$$Y(z)=W(z)+z^{-1}W(z)$$
$$Y(z)=W(z)\left(1+z^{-1}\right)$$
$$W(z)=\frac{Y(z)}{1+z^{-1}}$$

$$\omega(n)=x(n)+2k\omega(n-1)$$
$$W(z)=X(z)+2kz^{-1}W(z)$$
$$X(z)=W(z)\left(1-2kz^{-1}\right)$$
$$W(z)=\frac{X(z)}{1-2kz^{-1}}$$

$$\frac{Y(z)}{1+z^{-1}}=\frac{X(z)}{1-2kz^{-1}}$$
$$\frac{Y(z)}{X(z)}=\frac{1+z^{-1}}{1-2kz^{-1}}$$
$$H(z)=\frac{1+z^{-1}}{1-2kz^{-1}}$$

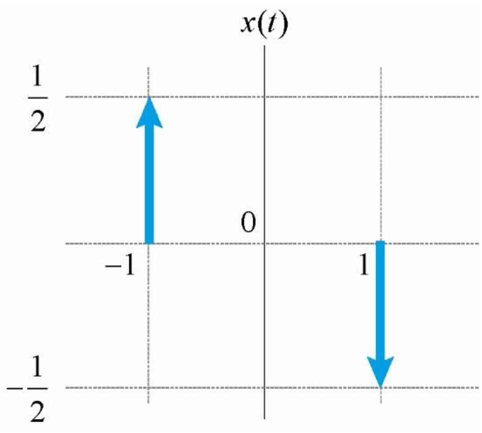
$$|YB|>2k<1$$
$$|2k|<1$$
$$-1<2k<1$$
$$\frac{-1}{2}<k<\frac{1}{2}$$

$$H(z)=\frac{1+z^{-1}}{1-2kz^{-1}} \qquad h(n)=? \qquad k=\frac{1}{4} \text{ aldığımızda koşul sağlanmış olur}$$

$$H(z)=\frac{1+z^{-1}}{1-2\left(\frac{1}{4}\right)z^{-1}}=\frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}} \qquad |z|>\frac{1}{2}$$

$$H(z)=\frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}}$$
$$=\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{z^{-1}}{1-\frac{1}{2}z^{-1}}$$
$$h(n)=\left(\frac{1}{2}\right)^{n}u(n)+\left(\frac{1}{2}\right)^{n-1}u(n-1)$$

BSM307 Örnek Final 6.Soru



$$X(\omega)=?$$

BSM307 Örnek Final 6.Soru Çözüm

$$X(\omega)=\int_{-\infty}^{\infty}\left(\frac{1}{2}\delta(t+1)-\frac{1}{2}\delta(t-1)\right)e^{-j\omega_0 t}dt$$

$$=\frac{1}{2}\int\delta(t+1)e^{-j\omega_0 t}dt-\frac{1}{2}\int\delta(t-1)e^{-j\omega_0 t}$$
$$\qquad t=-1 \qquad\qquad t=1$$

$$=\frac{1}{2}e^{j\omega}-\frac{1}{2}e^{-j\omega}$$

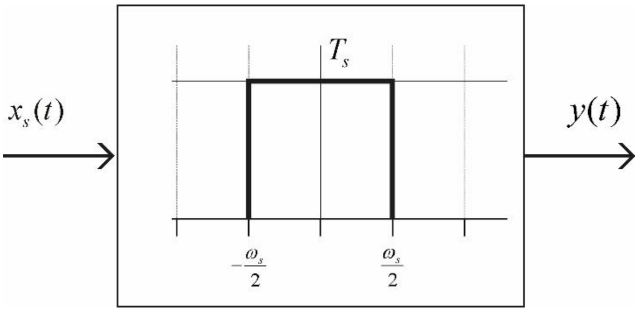
$$=\frac{e^{j\omega}-e^{-j\omega}}{2}$$

$$=j\frac{e^{j\omega}-e^{-j\omega}}{2j}$$

$$=j\sin(\omega)$$

BSM307 Örnek Final 7.Soru

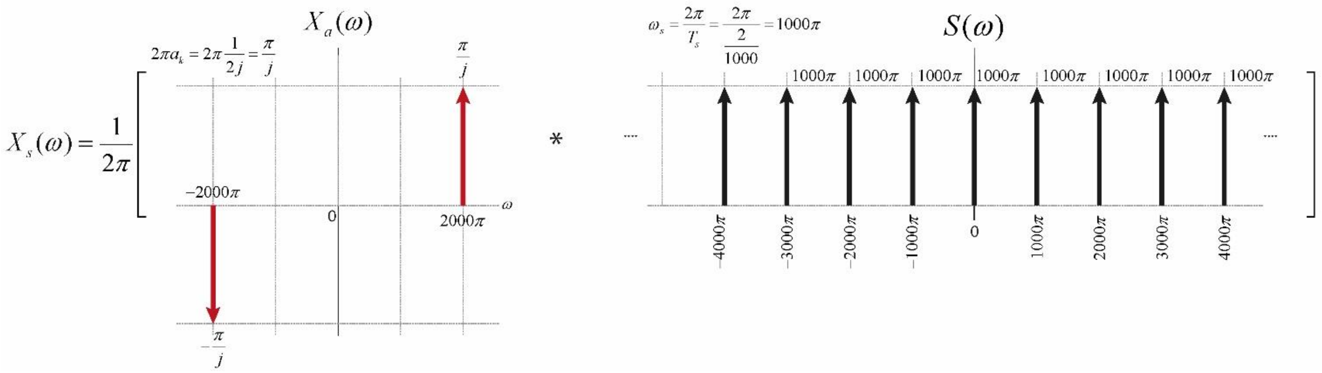
$x_a(t) = \sin(2000\pi t) \quad T_s = 2ms \quad X_s(\omega) = ? \quad Y(\omega) = ? \quad y(t) = ?$

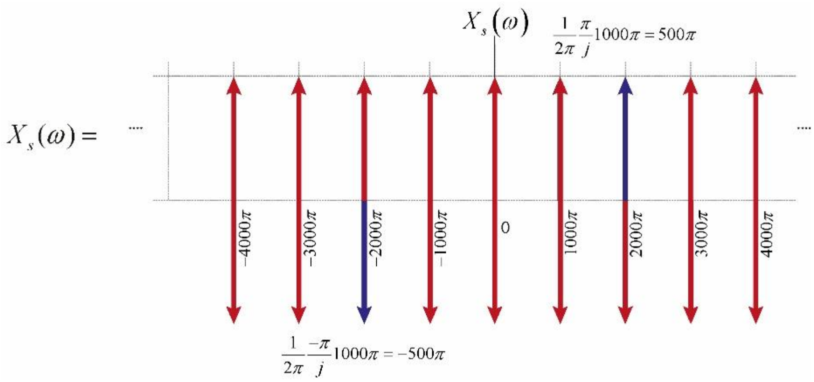


BSM307 Örnek Final 7.Soru Çözüm

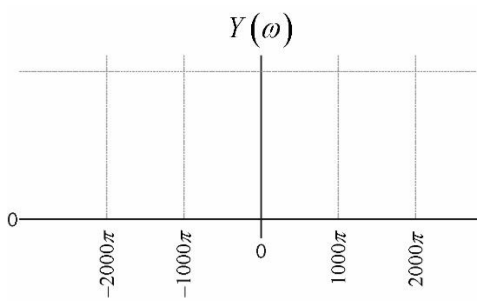
$$T_s = 2ms = \frac{2}{1000}s = 2000Hz$$

$$\omega_0 = 2000\pi \qquad \omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{\frac{2}{1000}} = 1000\pi \qquad 2\pi a_k = 2\pi\frac{1}{2j} = \frac{\pi}{j}$$





$$Xs(\omega) = 0$$
$$Y(\omega) = 0$$
$$y(t) = 0$$



Bu döküman

Seçkin ARI hocanın | İşaretler ve Sistemler 2017 Yaz Okulu dersinde anlattığı ve tahtaya çözdüğü örneklerden oluşturulmuştur. Dökümanı istediğiniz gibi kopyalayabilir dağıtabilirsiniz. Bazı örnekler hocanın kendi verdiği sunularda olduğu için oradan kopyalanmıştır. Hangi dökümandan kopyalandığı örnekte belirtilmiştir. Faydalı olması dileğiyle, doküman içerisinde hata olduğunu düşünüyorsanız bulentaltinbas@hotmail.com adresine mail atarsanız sevinirim.

Sakarya Üniversitesi
Bilgisayar Mühendisliği
Bülent ALTINBAŞ
14.07.2017